European Proceedings of
Social and Behavioural Sciences
EpSBS
www.europeanproceedings.com
e-ISSN: 2357-1330

DOI: 10.15405/epsbs.2021.05.264

ISCKMC 2020
International Scientific Congress «KNOWLEDGE, MAN AND CIVILIZATION»

MATHEMATICAL PROGRAMMING IN MANAGEMENT: METHODS AND MODELS

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Abstract

The paper provides an overview of the impact and use of mathematical programming in management. Mathematical programming is currently used in completely different dimensions. The paper evaluates the existing methods of mathematical programming and optimization as a management toolapplicable for developing and solving nonlinear programming tasks in management. It is proved that management as a sphere of science uses mathematical models to give managers particular guidelines for effective decision-making using available or additional information, if existing knowledge is not enough to make an optimal right decision. Various classification methods for optimization methods and models are proposed. Mathematical programming in management is useful since it provides possibility to obtain information about qualitative properties and quantitative characteristics of a target object avoiding field experiments. This may justify the expense required for overcoming difficulties that arise in development stage or when trying to use mathematical models. That is, the main difficulty that one has to face in mathematical programming is to make sure that this model is adequate to the target object. The authors believe that it is necessary to find out how accurately this model reflects a real situation and how reliable the quantitative estimates can be with this model.

Keywords: Mathematical programming, management, simplex method, optimization

https://doi.org/10.15405/epsbs.2021.05.264
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Selection and peer-review under responsibility of the Organizing Committee of the conference
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1. Introduction

In the 21st century, a considerable part of sciences, in one way or another, are linked with mathematical programming. Management science (management) is no exception. A scientific approach to make management decisions is a characteristic feature of management.

However, this problem appeared back in early civilizations. Only during World War II it started to be considered as a well-regarded and separate field of knowledge. Since that period, it had grown at a furious pace, never before experienced in comparison with other scientific breakthroughs. Management changes the way we perceive decision making and permeates every imaginable application, spanning various issues such as public, industrial, business and military (Gelikh, 2004).

Management has many other manifestations. Sometimes the scientific approach to management issues is determined by such terms as cost-benefit analysis, systems analysis and cost-effectiveness analysis.

One of the most developed and widely used management branches is mathematical programming, namely linear programming.

This branch of management deals with limited resources and optimal distribution of them between competitors according to a number of restrictions connected with the nature of research issue. These restrictions may be caused by technological, financial, organizational, marketing or other reasons.

In general, mathematical programming is a mathematical representation the purpose of which is programming or planning of the best existing way to use and distribute limited resources. In case when mathematical representation is based only on linear functions, it is a model of linear programming (Artemovsky, 2018).

George Bernard Dantzig is the author of simplex method developed for the solution of the problem of general linear programming. He developed this method in 1947.

He was in the team of the US Air Force Research Group or Scientific Computing of Optimal Programs (SCOOP).

Due to unprecedented computational efficiency, reliability of this method and the access to digital computers with high speed it was possible to transform linear programming to the most powerful method of optimization and the most popular one in the context of business (Gorbach & Shakhov, 2018).

2. Problem Statement

Mathematical programming is only a tool in management in general. That is why first of all it is necessary to get acquainted with a management and scientific approach and define the place of mathematical programming in it.

It is difficult to give an unambiguous definition of management. The field benefits from impacts in different sciences (social, natural, econometrics, mathematics, etc.) many of which avoid giving rigid definitions.

Management as a sphere of science uses mathematical models to give managers particular guidelines for effective decision-making using available or additional information, if existing knowledge is not enough to make an optimal right decision.

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3. Research Questions

This statement has several elements and we pay due attention to them.

Firstly, the idea of management science is behind an approach to building a model. Thus, it presents the attempt to cover the most important components of a target solution through mathematical abstraction. What is more, these components should give a complex and real representation of the decision-making process and include the elements characterizing the gist of the research issue. This task is rather difficult challenging. However if it is performed correctly, it provides a huge tool which can be used in complicated situations where it is necessary to make a decision.

Secondly, using these efforts, management may give leaders particular guidance or improve the understanding of managers of probable effects caused by their actions.

4. Purpose of the Study

The main purpose of this research is to support management actions. According to the authors the crucial role in successful management approach is played by the complex of model-provided great power of a computer and insightful decision maker.

5. Research Methods

A theoretical and methodological base is systemology (principles of system approach and methods of systems analysis) and classification of methods and models of mathematical programming, theory of complex systems, theory of optimal decision making, modeling technology, methodology of object-oriented design (programming), methods of linear and nonlinear programming.

6. Findings

Finally, it is the complexity of a target decision, rather than a tool used to study the process of decision-making, that determines the information required for effective decision-making. Models have been criticized for that they place unreasonable demands to information. This is not really necessary. Conversely, models can be built subject to the current status of information available. Moreover, the models are used to assess the financial relevance of the collection of additional information.

In scientific literature there are several approaches for the classification of models. To start with, we will describe a categorization. It determines extended types of models based on the degree of realism during the presentation of a given problem:

1. The operational exercise is the first type of model. This modelling approach is related to the real decision environment. Modelling process simply involves the development of a series of experiments in the context of this environment and measurement and interpretation of the results of this research. For example, it is necessary to determine which blend of several crude oils should be mixed in oil production in order to meet the market requirements of supplied final products. If we conduct the operational task to support the solution, we will try different quantities of crude oil combinations directly in this production process and estimate the revenues and costs of every combination. After the series of attempts, we will

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understand the relationship between the crude oil input and the net income received during the refining process, which will allow determining the appropriate combination (Mazalov & Chirkova, 2018).

For this approach to work successfully, experiments should be carefully planned, the results of the experiment should be evaluated in the light of errors that might be caused by inaccurate measurements, and inferences about decisions made should be drawn based on a limited number of observations. The operational task idea presents the inductive learning process. This process is typical of practical experiments in some natural sciences. In these scientific fields the generalizations are made on the basis of specific observations of a particular phenomenon.

These operational tasks are highly realistic in any kind of modelling approach. It can be explained by the fact that only some external abstractions or simplifications are included in it in comparison with those related to the explanation of obtained results as well as guidelines and generalizations. However, sometimes this method is extremely expensive. In addition, often a person who makes a decision is not able to perform an extended analysis of available alternatives. As a result, this can entail serious sub-optimization in final conclusions (Bishara, 2018).

2. Gaming is the second type of modelling in this classification. Here a model is developed as abstract and simplified variant of reality. The model gives a response for the evaluation of the effectiveness of alternatives being proposed that a decision-maker provides in consistent and organized way.

This model should reflect the proportion between input and output of refining with an acceptable degree of accuracy. Subsequently, all personnel involved in arrenging the decision-making process will have an opportunity to operate with the model. A production manager will draw up the plans of production, a marketing manager will make contracts and develop strategies and a purchasing manager will determine crude oil prices and sources and create procurement programs etc. Certainly, we will lose some degree of realism in this modelling approach in relation to operational tasks, because we deal with abstract situation, but we retain some of the human interactions of real process (Mazalov & Chirkova, 2018). Despite the fact that the processing cost of each alternative is decreasing, the rate of performance measurement is increasing.

Games are used primarily as a teaching instrument to develop an awareness of complexities inherent in decision-making.

3. Simulation models are like game models. The only exception is that decision-makers do not participate in the process of simulation. The model gives tools for the assessment of the effectiveness of many alternatives externally provided by a decision maker, without consideration of human interaction at modelling intermediate stages.

Similar to operational exercises and games, simulation models do not generate alternatives and do not provide an optimal answer to the target solution. These types are presented by inductive and empirical models. They are useful for the assessment of the effectiveness of alternatives which a decision-maker identifies in advance.

Using a simulation model in the refinery example, many combinations of quantities and crude oil types would be pre-programmed and the net profit connected with every alternative would be generated,

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without any external cost incurred by decision-makers. The obtained results of the model could serve as a basis for new experiments until they had reached the correct problem understanding.

Various models of simulation acquire the form of computer programs. The logical arithmetic operations in them are arranged in predetermined order. Therefore, there is no need to determine the problem in pure analytical terms.

4. This model category is analytical. Here the problem is entirely mathematical. The model calculates the optimal solution. All constraints are satisfied and the best possible value for this objective function is provided by this solution.

In oil producing example, the use of the analytical model means setting the goal of maximizing the net income. In turn it is generated from the refinery based on crude oil and its types and quantities. The solution will be the exact amount of every available type of processed crude oil, which will increase the net profit for maximum within the existing set of constraints.

In the past two decades, linear programming has been an undeniable analytical model exploited to solve such problems. In general, the analytical models are the cheapest and easiest in their development. Nevertheless, they bring the highest simplification in model representation.

In order to get a general problem statement of mathematical programming, it is necessary to formally represent the general model of linear programming.

In mathematical context, a linear programming model is considered as maximization (or minimization) of objective function with certain linear constraints. In particular, we can describe a problem of linear programming as the search of the values of n $x_1, x_2, .., x_n,$ decision variables, so that they maximize (or minimize) Z objective function

Where:

$$Z = c_1x_1 + c_2x_2 + \cdots + c_nx_n \rightarrow extr \quad (1)$$

Taking into account the following restrictions:

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n &\le b_1, \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n &\le b_2, \\ &\vdots \qquad\qquad \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n &\le b_m \end{aligned} \quad (2)$$

where basically:

$$x_1 \ge 0, x_2 \ge 0, \cdots, x_n \ge 0 \quad (3)$$

a c_j, a_{ij} and b_i are constant.

It is easy to interpret a general problem of linear programming stated above in production problem context. For example, we may assume that this production facility does not have *n* possible products that can be produced. For every product, we want to determine the production level, denoted by $x_1, x_2, .., x_n$. Moreover, these products compete for limited resources denoted by *m* that can comprise available labor, machine capacity, demand for products, working capital, etc., and are denoted as $b_1, b_2, .., b_m$. Let a_{ij} be the amount of i resource (where i varies from 1 to m) that is required for

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j product (where j varies from 1 to n), and let c_j be the unit of profit for product j. Next the linear programming model finds the output of every product to maximize the total net profit z (1), taking into account that we must not exceed the available resources (2) and that only positive or zero quantities of products can be produced (3).

Linear programming is presented not only through the example of the above mentioned structure. Firstly, we can minimize rather than maximize the objective function. Moreover, the "≥" or "=" constraints may be processed concurrently with the "≤" constraints (presented in constraints 2). Finally, particular variables can have both positive and negative values.

Certain technical definitions are connected with programming that needs to be defined in more precise terms. The set of solution variables values, described by constraints (2) and (3), form the admissible domain of a problem. A possible solution further optimizing the objective function (1) is considered as the best possible solution.

The solution of a linear program can lead to 3 possible cases:

1. A linear program may not be feasible, which means that there are no values of $\mathrm{x_1, x_2, .., x_n,}$ decision variables satisfying all the constraints (2) and (3) simultaneously.

2. Unbounded solution: if we maximize, the value of the objective function can indefinitely increase and does not break any constraints. (If we minimize, the value of the objective function can infinitely decrease).

3. Generally, it will have at least 1 final optimal solution, and often there will be several optimal solutions.

The simplex method developed in order to solve linear programs gives an efficient algorithm for the construction of optimal solution, if it exists, or for the determination of infeasibility or unbounded nature of a problem.

Notably, in linear programming statement, the variables of decision can take any continuous value. For example, the values such as $\mathrm{x_i}$ will take real values are perfectly acceptable if they satisfy the constraints (2) and (3). The important extension of this model is the requirement that all (or some) of the variables of decision are limited to integer values.

One more important extension of the given model is the ability of the objective function, constraints (or both of them) to become non-linear functions. The general model can be perceived as the search of the values of $\mathrm{x_1, x_2, .., x_n,}$ variables of decision, that maximize (or minimize) Z objective function

Where:

$$Z = f\,(x_1, x_2, .., x_n) \rightarrow extr \tag{4}$$

According to the following restrictions:

$$\begin{aligned} f_1\,(x_1, x_2, .., x_n) &\le b_1, \\ f_2\,(x_1, x_2, .., x_n) &\le b_2, \\ \vdots \qquad & \qquad \vdots \\ f_m\,(x_1, x_2, .., x_n) &\le b_m \end{aligned} \tag{5}$$

where:

$$x_1 \ge 0, x_2 \ge 0, \cdots, x_n \ge 0 \tag{6}$$

For maximized objective function, inequalities are written with the sign "less than or equal to", and for minimized – with the sign "greater than or equal to".

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In nonlinear programming, the values of a right-hand side are often incorporated in the definition of a function $f_i\ (x_1, x_2, .., x_n)$, with a right-hand side equal to zero. For the purpose of the solution of the problem of nonlinear programming, it is necessary to make some assumptions on the form and behavior of the involved functions. The nonlinear functions need to be good enough so they have a computationally efficient means for the solution of a particular problem.

Optimization models can obey different classifications in accordance with the point of view we have adopted. Based on time periods studied in the model, the models of optimization can be divided into static with 1 time period or multistage with multiple periods of time.

Dynamic programming is one of the approaches to solving multistage problems. In addition, nowadays in the sphere of extensive linear programming, significant research work is being carried out on the development of special algorithms for solving multistage problems (Moskvitin, 2018).

One more important way to classify the models of optimization relates to the behaviour of the parameters of a model. The model of optimization is called deterministic provided that the parameters are known constants.

The model of optimization is called stochastic provided that the parameters are indicated as indefinite quantities, the values of which are marked by the distributions of probability. Finally, the optimization model is called parametric in case when particular parameters are allowed changing systematically, with the changes to be determined in the optimal solution complying with the changes of parameters. In general, such mathematical programming as stochastic and parametric pose more sophisticated problems than deterministic one. The deterministic modelling can be effectively used to solve complex problems up to 5000 lines and infinite number of variables. Moreover, after a deterministic optimal solution is obtained we can perform sensitivity analysis and parametric programming in linear programming.

The 3rd way to classify the models of optimization is related to the variables behaviour in optimal solution. If the variables take any value satisfying the constraints, it is continuous model of optimization. The model of optimization is called integer or discrete if the variables take only discrete values. Finally, when the problem has some integer variables and some continuous variables it is mixed optimization model.

7. Conclusion

In conclusion it is necessary to state that in general the problems with integer variables are much more challenging than the problems with continuous variables. The models of network present the type of the models of linear programming that is the exception to this rule because their special structure leads to optimal solutions of integer character. Despite a considerable progress in the field of linear programming of mixed and integer type, there is still no unified method for the efficient solution of all general medium-sized linear programs of integer nature in a reasonable time, although adequate computational methods have been developed for specific problems.

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