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ASSESSING VALUE WITH THE NEW METHOD OF STOCHASTIC RISK FACTORS

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Abstract

The paper examines the hypothesis about the rationality of investors in the light of behavioristic approach and propose three forms - strong, medium and weak form. Hypotheses are ranked depending on the ability of investors to predict risks and cash flows. The hypothesis in a strong form is applicable to fixed income with extremely low risk of default, in the medium form - for analysis of investment projects and the choice of capital structure. Rationality in the weak form is applicable to the analysis of risky assets - shares. The weak rationality of investors relates rather to the behavioristic economy, as it is based on intuitive perception of real behavior of investors in the asset valuation. For the purposes of modelling and analysis, there proposed the concept of stochastic risk factors, which comes from the common stochastic discounting factors, but is different in the basic ideas. Particularly, it is applied to firms and the risks are reflected only in discount rate, but not in cash flows. Two basic theorems are proven - about the separation of cash flows to the basic and growth projects and the conditions for the economic efficiency of the growth project. To substantiate the stochastic risk model, a valuation model proposed with short-term and long-term trends. Both trends indicate risks and may change unpredictably. As the result it is confirmed that the model of stochastic risk factors is applicable and consistent.

Keywords: Behavioristic economics, investor rationality, long-term trend, stochastic discounting factors, stochastic risk factors

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1. Introduction

Modern economic theory is based on the hypothesis of rational behavior of economic agents. This behavior, according to a mainstream theory, manifests itself in the choice of economic business models, commodity and securities prices, investment strategies and even risk assessment. Among other things, the hypothesis about rational assumptions of the investor is the key to the mainstream financial theory, including CAPM, CCAPM, ICAPM, MM theories, the Black-Scholes theory and all their modifications, as well as the theory of macroeconomic equilibrium. Tirole (2006) notes that from a fundamental point of view the theory CCARM and MM may be obtained as a consequence of the most fundamental theory of economic equilibrium by Arrow – Debreu (Arrow, 1986) with some additional assumptions.

However, now it seems apparent that hypotheses of rationality and especially the hypothesis of rational expectations of investors are poorly confirmed by practice and are therefore subject to reasonable criticism from practically minded economists, including Stiglitz et al. (2019), Shiller (2008) and Thaler (2015). A complete rejection of the hypothesis about the rationality of investors would turn the securities market into a casino. In this case, the investor's profitability may be truly considered as martingale. But this approach is doubtful - it is well known that investors constantly analyze assets and associated risks and predict changes in prices and cash flows.

There is further assumed that in most cases violation of the hypothesis of rational expectations of investors is not related to the irrationality itself or subjectivity, but to the unreliability of forecasts and frequency they have to be revised. Investor forecasts for the future value of financial assets, taking into account their risk, may be reflected in projected cash flows or in the discount rates applied to them. In theory, both approaches are equivalent because the discount operation is equivalent to a reduction in expected cash flow. In the classical theory of finance, expected cash flows are traditionally seen as cash flow with a constant long-term trend and a normally distributed stochastic error, and the discount rate in MM is usually determined by the cost of capital at the current time. On the other hand, it becomes apparent that the risks that investors take into account are not always reflected in the WACC or the value of equity (Zhukov, 2018, 2019). The issue how investors estimates future value is insignificant in terms of the classical definition of market information effectiveness. However, this issue is important from the point of view of research based on the analysis of investor behavior, i.e. behavioral economics and so for the current research.

The current understanding of SDF models is described in the Campbell review of the modern achievements in the theory of finances (Campbell, 2014) and it is: consider the model with the S states of economy s 1, ...S, the probability of each has a strictly positive value p (s). Suppose the market is complete, which means that for each state there is a conditional obligation with a payment of $1 in the state of s and 0 otherwise. Let's write down the price of this conditional security as q(s). Then any asset may be presented in the form of conditional (depending on the state of the economy) payments X (s). The law of one price (LOOP) states that two assets with the same payments in each state s=1, ... S must have the same price. Next, the price of an asset may be presented as:

$P(X) = \sum s=1,S\ q(s)\ X(s)$

Multiplying and dividing the price of conditional liabilities q(s)by the probability of the state of π (s), get the basic identity for stochastic discount factors:

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$$P(X)=\sum s=1,S\ \pi(s)(q(s)/\pi(s))X(s)=\sum s=1,S\ \pi(s)M(s)X(s)=E[MX] \quad (1)$$

The M(s) by definition is the stochastic discount factor (SDF). The (1) may be written for N periods and even for infinite horizon. The most common use of SDF is to justify the impact of changes in consumption on the value of gross or industrial indexes based on consumption CAPM theory - CCAPM (Favero et al., 2019). At the same time, changes in the company's value are commonly treated as martingale with a constant long-term trend (Qin & Linetsky, 2017). Proponents of this approach, based on classical financial theory and concept of information market efficiency insist that even data on price fluctuations caused by the uncertainty of the price model (which contradict this approach) can be explained with that theory (Xu & Yao, 2019), and the latest – (Hansen et al., 2020). However, results of the mainstream theory of finance, based on "perfection", "completeness", "efficiency", "equilibrium" and especially "fairness" and "benevolence" of financial markets, is the subject of increasing criticism, e.g. by Stiglitz et al. (2019), Shiller (2008) and particularly Thaler (2015), who proposed the new paradigm – behavioristic economics. It is considered to be fundamentally different from the mainstream theory, as it is based on the analysis of the real behavior of the investor and does not accept artificial and unrealistic assumptions of classical theory. However, behavioristic economics needs to do much more to construct new formal models and methods, independent on the concept of the "fair and perfect" market and this is particularly the aim of this work.

2. Problem Statement

The goal is to construct such hypotheses of rationality of investors, that are related to the intuitively simple perception of investor's approach. And there should be hypothesis of the rationality of investors in the most general (weak) form, strong form and intermediate form. Then, a new model for the valuation should be built. As it is assumed to be based on an intuitive perception of the investor's behavior (so unrelated to the mainstream financial theory), it will belong rather to behavioristic economics. The next goal is - to propose exact and formal models for rationality of investors and to check its feasibility. Further, there arises the task to check its applicability for analysis and forecasting of the value. To classify the rational expectations of the investor, the first reasonable intuitive assumption is - they should differ in how predictable future cash flows and risks are. In a term of forecasting and revision of forecasts, the following three forms of investor rationality may be separated:

- the strong, when an investor predicts cash flows and discount rates for an infinite period of time, without major changing of forecasts;
- the intermediate, when an investor is able to do it only for a limited investment horizon (e.g. year, quarter, etc., so the longer the period, the higher is the assumed rationality of investor);
- the weak when investor permanently revises forecasts of cash flows and discount rates, based on new incoming information.

3. Research Questions

To describe the weak form of rationality of investors, based on the definition of SDF (1) there will be introduced the concept of stochastic risk factor SRF (Zhukov, 2021) which differs from the usual SDF.

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Particularly, SRF are applied to expected cash flows from the firm. Thus, the main risks are reflected in the discount factors, but not cash flows. That approach comes from outstanding empirical results of Cochrane (2011), which states, that volatility of value is created largely by the volatility of discount rates.

Definition: SRF is a stochastic process of {Mτ,τ+t}such that with expected payoffs Eτ CF(τ+t), the value of asset at the time of the valuation τ is:

$$V(\tau) = \sum t=1,\infty M(\tau,\tau+t)\ E\tau\ (CF(\tau+t)) \quad (2)$$

Here, τ – the point of assessment, the Eτ - Boreal measure (probability), depending on the moment of assessment. Note that the definition of SRF (2) differs from the definition of SDF (1) in many ways, but one is the simplest - discount rates in (2) are applied to expected cash flows (equivalent to deterministic). So, SRF match risks and volatility of value following results of Cochrane (2011). Recombining (2) go to the hypothesis about rational expectations of investors in the most general form (Zhukov, 2019):

$$V(\tau)=\sum t=1, \infty\ ECF(t,\tau)/(1+r(t,\tau))t \quad (3)$$

$$ECF(t,\tau) = E\tau\ (CF(\tau+t)),\ 1/(1+r(t,\tau))t = M(\tau,\tau+t)$$

This equality assumes that discount rates may change over time t and, among other things, the possibility of hyperbolic discounting. And in the case of application (3) for capitalization (but not enterprise value) zero-period cash flow (cash and marketable securities) should be added. The model (3) may be also obtained from the simplest two-period model (1), or from any model for N periods (assuming that for infinite N residual part of (3) tends to zero). One can also get any model for N periods from (3) assuming cash flows are zero from the moment N+1.

Then, for the least “rational” investor, whose forecast is limited to one period and may be instantly revised, (3) takes the form:

$$V(\tau)=ECF(1,\tau)/(1+r(1,\tau))+V(1,\tau)/(1+r(1,\tau)) \quad (4)$$

Investor benefits are considered as stochastic, while distribution and variance are generally considered unknown. But mathematical expectation of cash flows must be predictable, for example, with a scenario model with S states similar to the model (1) for SDF. Otherwise, no model for asset valuation may be applied and stock market would turn into casino. That may be even good for mainstream theory, because in that case value really will follow martingale. But that is hardly realistic – it is well known that investors forecasts cash flows in the assessment.

The next step to basic SRF model is to introduce the concept of the basic cash flow, corresponding to the cash flow to the firm with zero growth rate and minimal investments.

Definition: Basic free cash flow is the free cash flow from the current business, provided the minimum investment ensures that the expected cash flows are unchanged (the condition of zero growth).

Formal expression for the underlying cash flow may be presented in the form (5):

$$FCF0(\tau)=arg\{EV(FCF(t,\tau))|\ EV(FCF(t,\tau))=EV0(\tau)\ \&\ dFCF(t,\tau)/dt=0\} \quad (5)$$

Decomposing the company's free cash flow into the base flow and flow of the investment development project the following theorems are proved (formal assumptions and proofs omitted):

Theorem 1. With the SRF model (3), the enterprise value may be presented as two components - the value of core business with zero growth and the net present value of the investment development project, which provides growth.

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Theorem 2. At the assumptions of the theorem 1 the enterprise value at the point of assessment "τ" may be presented in the basic form (6) as the production of basic free cash flow (free cash flow at zero growth) and stochastic risk factor SRF, the reverse difference of the required return on capital "r(τ)" and average long term growth rate "g(τ)".

$$EV(\tau) = ECF0(\tau)\ SRF\ (\tau) \quad (6)$$

$$SRF\ (\tau) = 1/(r(\tau) - g(\tau))$$

Corollary. In the conditions of the theorem 1, a firm may be presented as a single project with an average investments CAPEX(τ), with the same cost of the capital r(τ) and with the same average growth rate of cash flows g(τ)*≥0 (higher than g(τ) in (6)). The development project will be effective if and only if:

$$CAPEX(\tau) \leq FCF0(\tau)\ g^*(\tau)/\ r(\tau)$$

Growth rate of cash flows g* (τ) may be evolved from (6) and CAPEX with the identity:

$$FCF0(\tau)/(r(\tau) - g(\tau)) = (FCF0(\tau)\text{-}CAPEX)/(r(\tau) - g^*(\tau))$$

Thus, research questions include:

1. The applicability of the basic SRF model (6) to analyze the value dynamics of companies and the cost forecast.

2. Is the long-term trend variability predictable?

3. Is the short-term trend variability for underlying cash flow predictable?

4. Purpose of the Study

The purpose of the empirical study is to check the applicability and consistency of the base model SRF (6). From the theorems 1 and 2 follows, that the value of a firm may be presented through the basic cash flow and stochastic risk factor (SRF). As proved in the section 3, the last is the reverse difference of the required return on capital "r(τ)" and average long-term growth rate "g(τ)".

Evidently, an investor is capable to assess the basic cash flow and required return on capital "r(τ)". Basic cash flow is just expected free cash flow to the firm under condition of minimal investment. And required return on capital "r(τ)" is just the rate of return which investors require under the given risk of the company. Commonly it is assessed through the beta, market risk premium and riskless rate, while the specific way to assess these parameters may be different. So, the main question is – could investor reliably predict the future assessment of the growth rate?

In the light of (6) it turns to the more familiar question - may investor reliably predict future value of a company by the current value? If that is true, then current enterprise value must be good predictor to the future enterprise value given there are no principal changes in the:

- underlying business model (and so basic cash flows),
- macroeconomic (systematic) risks (such as financial crisis or something similar),
- individual (idiosyncratic or systemic) risks of the firm (such as financial instability or default).

For the basic model (6) this question may be stated in the simplest form – how accurately can investors assess the future value of a company (so, is the present value good predictor for the future)? And if answer is positive, it is necessary to estimate accuracy of that prediction.

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Basing on the parameters of the model (6), the positive answer would give reliable confirmation that investors are quite good at predicting the average growth rate of the company. With the model (6) it is equivalent to prediction future value of a firm with the current value.

5. Research Methods

Assuming a short-term growth of the basic cash flow in (6) is equal g*(t), the basic SRF model (6), may be rearranged into empirical SRF model with two growth rates:

$$EV(t)=FCF0 \exp (g^*(t)\, t)\, SRF(t) \quad (7)$$

$$SRF(t) = 1/(r(t) - g(t))$$

Here in (7) r(t) is the cost of the capital (assumed to be assessed on the basic of opportunity costs) and g*(t) is the short-term trend. Both are assumed to be conditionally constant at short intervals of one period, as all the main risk factors are supposed to be accounted in the long-term growth rate g (t) (long-term trend), so in the stochastic risk factors (SRF(t)).

To the addition of the main research questions pointed at the section 3 and 4, following future discussion questions may be subject for research with the model (7):

1. How accurately can investors predict the basic cash flow FCF0, cost of capital r(t), and the long-term growth trend g(t)?

2. How accurate may be assessment for the changes in the short-term growth trend g*(t)?

3. Can investors predict changes in SRF(t) risk factors?

Presumably from the model (6) investor should be able to predict the underlying parameters of (6) - FCF0, g(t), and cost of capital r(t), so they should be able to predict future value of the company EV. And then current value EV (t) must be very good predictor for the future one EV(t+1).

However, regarding questions 2 and 3 assumptions are rather negative – as the both reflects unpredictable risks - g*(t) short term and SRF(t) – long term (through the long-term growth rate g (t)). To answer these questions, consider the next econometric model (8). From the (7) the following econometric model may be evolved:

$$Ln (EV(t+1)) - Ln (EV(t)) = g^*(t) + Ln (SRF(t+1)/SRF(t)) \quad (8)$$

For the assessing stochastic risk factors it is possible to enter in the model (8) an additional parameters g*(t) and SRF(t) (internal or external), responsible for changes in trends - long-term g(t) (in the SRF(t)) and short-term g*(t). The optimal values for those may be found with GMM (generalized method of moments). However, to confirm the significance and applicability of the core SRF model (6), apparently it is sufficient to treat (6)-(8) with the next simplest autocorrelation model (9) with OLS:

$$Ln (EV(t+1)) = A\, Ln (EV(t)) + B + eps \quad (9)$$

Here "A" shows the accuracy of the forecast of future EV at the moment of t, B is equal to g*(t), and the eps is a random error associated with changes in the stochastic risk factor SRF (so in g(t)) plus errors in the estimation of the short-term growth rate of g*(t).

6. Findings

The question of unit root for (9) is evident - as "A" is always smaller than 1, the autocorrelation model doesn't have the unit root 1. Model (9) evaluations conducted for companies from a wide variety of

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industries (partially presented in Table 1) show a high determination of regression (value is usually lower than E-08). A very high level of R2 is evident, which indicates, that future value is predictable at the current date by investor with high confidence. And the main result is the high value of accuracy of prediction "A" at 0.7-0.95, with the very low p-value (of the insignificance of this parameter) and the very high R2 ratio. However, the factor B, which indicates the short-term trend g*(t) and plus changes in the long-term trend g(t) is usually not significant (as expected). Also, standard errors for B may be unacceptably large. All that means - the combined effect of changes in the short-term trend g*(t) and the long-term trend g(t) are typically unpredictable (as assumed).

Table 1. Check for SRF Model (9) with OLS statistics.*.

Firm	BP	Coca-Cola	Shell	Apple**	Facebook**	Wall-Mart
A	0,72	0,95	0,92	0,84	0,97	0,88
A- stand. err.	0,09	0,04	0,055	0,09	0,05	0,047
B	3,19	0,248	0,93	2,03	0,5	1,33
B- stand. err.	0,085	0,22	0,66	1,27	0,7	0,48
Sign. of regr. F (p-value)	5,08E-12	3,01E-26	2,56E-25	2,9E-08	5,47E-13	4,12E-29
Sign. for A (p-value)	5,08E-12	3,01E-26	2,56E-25	2,9E-08	5,47E-13	4,12E-29
Sign. for B (p-value)	0,002	0,258	0,16	0,122	0,48	0,02
R2	0,72	0,95	0,9	0,89	0,96	0,916
R2 - stand. err.	0,116	0,08	0,126	0,15	0,2	0,2
Sample	66	49	68	22	22	71

* Available (partial) quarterly data for 2000-2019.

** Partial annual data for 2000-2019.

Source: author.

So, the results of the test for the simplest SRF model (9) shown in Table 1 demonstrate, that the basic SRF model (6) based on an assessment of future growth rates generally explains investor valuations well. To a certain extent, this reflects a well-known fact - the past value of the company is usually a good predictor for the future. However, in the context of the theorems proved in the section 3 and research questions which were set up, this means – the basic model (6) is substantial and feasible to apply. However, probably, exception must be reserved for crisis situations and sharp change in trends.

It should be specially noted that the results of calculations with the simple model (9) do not refute and do not confirm the conclusions of the classical theory (Hansen et al., 2020), considering the price of the company as a martingale type stochastic process added to determined trend (this was not the purpose of the study). Intuitively, this hypothesis may seems as not very realistic, but it might be true and so it needs additional checks. Perhaps the econometric SRF model (8) which captures dynamics of both short- and long-term trends, may be used to confirm or deny it.

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7. Conclusion

1. The company's cash flows may be separated into two projects (theorem 1) – a basic business project with zero growth rate and a growth project. The basic business project is relatively stable and is based at the expected free cash flow at the zero-growth rate.

2. The growth project may provide positive long-term growth rate in the case of positive net present value and negative otherwise.

3. For the firm may be constructed equivalent investment project with the average investments CAPEX(τ), the same cost of the capital r(τ) and the same average growth rate of cash flows g(τ)≥0. Then development project will be effective if and only if: CAPEX(τ) ≤ FCF0(τ) g(τ)/ r(τ)

4. The results of the calculations presented in the previous section on the model (9) provide reliable confirmation that investors are quite good at predicting the future estimate of the average growth rate of the company at current date. That answers positively research questions, pointed at section 3.

5. Empirical results prove the applicability and consistency of the base SRF model (6).

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