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MATHEMATICAL EDUCATION CONTENT IN THE CONTEXT OF MATHEMATICAL LITERACY

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Abstract

The article discusses the search for a mechanism to update the content of mathematical education necessary to solve the problem of formation of mathematical literacy of graduates of primary school. The analysis of the state educational standards of the first and second generations, control measuring materials for carrying out the state final certification in mathematics in the context of results of the international monitoring research PISA (Programme for International Student Assessment) in which functional literacy of pupils in the field of mathematics is estimated, and also conclusions of the Russian national researches of quality of mathematical education is carried out. A link between the standardization of common education, changes in the content of mathematical education from one side and the improvement of the results of the Russian participants of PISA in the last cycles of this monitoring research from another side is established. It is proposed to allocate additional components in the content of education which are “Logic”» to ensure the development of skills of logical reasoning, “Mathematics and the outside world” to ensure the ability to apply the mathematical apparatus in real situations, and “Mathematics in the science and technology”, responsible for the formation of interdisciplinary connections and experience in the application of mathematics to the study of other school subjects. The areas for improvement of state examination materials are shown.

Keywords: Mathematical literacy, mathematics education content.

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1. Introduction

Currently the main focus of the educational system is necessityto form the ability of Russian schoolchildren to solve practical oriented tasks. The main impulse to startto discussthe mathematical literacy problem was the conducting of the monitory study PISA (Program for International Student Assessment), which assesses the functional literacy of students in the field of natural science, mathematics and reading. It turned out that Russian students do not know how to apply their mathematical knowledge in real life situations (Results of the international study PISA 2015, 2016).

At the same time, the high-level subject education was recorded by another international monitory study - TIMSS (Trends in Mathematics and Science Study) (Results of the international study TIMSS 2015, grade 8, 2016).

2. Problem Statement

In the modern time the gap between the formed subject competencies and the lack of functional literacy becomes one of the main problems of secondary mathematical education. One of the evident explanations of these is that the traditional academic nature of the mathematic course, which has focus on the fundamental mathematical education of school graduates.

At the same time, the academic nature of school education is consistent with the theories of evolving education developed by Russian psychologists, one of the provisions of which is the determining role of theoretical knowledge in learning. The challenge, therefore, is to maintain a high theoretical level and to teach students how to apply knowledge in real-world situations as well.

In this regard, it is natural to search for ways that would increase the application role in the structure of mathematical education of students without theoretical component loss.

3. Research Questions

To find a mechanism to solve this problem, it is advisable to answer the following questions: whether the results of PISA were updated due to the renew of mathematical education content in the framework of Secondary education standardization and changes in the system of state final certification carried out in the last 15 years in Russia; what way is the best to direct the development of the main documents that determine the content of mathematical education.

4. Purpose of the Study

The purposes of the study are to analyze the changes in the regulatory sphere that have occurred since the early 2000s, to identify the nature of their input on the structure of mathematical education of students and to show the prerequisites for the development of practice-oriented component.

5. Research Methods

analysis of legal documents that determine the content of Secondary mathematical education; the state final certification content, conceptual documents of the international study PISA.

6. Findings

The application of mathematical knowledge as a goal of secondary education and as a problem of learning has long been discussed by specialists in the field of mathematical education. E.g., in the work of 1948 "Principles of selection and compilation of arithmetic problems" corresponding member of the Academy of pedagogical sciences Arnold (2014) give a deep analysis in terms of practice-oriented education of the so-called text tasks. A deeper excursion into history shows that the first collections of tasks written by the Egyptians on papyrus and Babylonians on clay tablets represent a description of real situations from the modern life at that time, first of all, various professional spheres (construction, taxes, agriculture, etc.), which were solved with the use of mathematics.

Since then, mathematics has come a long way and the problem is that what of this historical heritage should be studied in the framework of Secondary education, which concepts are necessary for modern man to be considered mathematically literate. In this regard, the question of teaching how to apply the knowledge in practice is a part of problem of the evolving of the mathematical education content.

According to the concept of the PISA study mentioned above, "Mathematical literacy is an individual's capacity to identify and understand the role that mathematics plays in the world, to make well-founded judgements and to use and engage with mathematics in ways that meet the needs of that individual's life as a constructive, concerned and reflective citizen" (OECD, 2006, n.p.). In other words, mathematical literacy is the ability to apply mathematics in different real – life contexts.

What problems are hidden behind the revealed inability of Russian students to apply what they own?

Arnold (2014) wrote:

> *The task of teaching students to master mathematical tools both in the sense of the ability to produce elementary operations, and in the sense of the ability to choose the right tool in the right case, is often replaced by a more easily achievable goal — to teach students to use these tools in a certain, regulated, sometimes quite complex sequence, according to certain established rules*. (p. 71)

70 years later and after the conclusions of the conducted in 2015 National study of the quality of mathematical education one of the explanations of the result is the following: "The ability to apply knowledge in practice was tested, in particular, with the help of tasks in which it was necessary to calculate the cost of tickets or purchases. In the standard situation, when there is no extra data and no additional conditions, about 60% of students cope with the task. At the same time, the emergence of additional information and the need to consider various payment options (family ticket, the action "three packages for the price of two") dramatically reduces the part of those who cope with the problem: 34% in 5 classes and 23% in 7 classes. And almost all students begin to perform these tasks, and the fall is due to the group of the strongest students" (Analytical materials on the results of the National study of the quality of mathematical education in grades 5-7. Part 2, 2015).

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It may seem that we are talking about the application of traditional mathematical content, however, practical experience and research show that the problem goes beyond the subject matter and subject skills (Roslova, 2018; Tyumeneva, Aleksandrova, & Goncharov, 2016; Andersen & Morgan, 2011).

The solution of the problem to form the mathematical literacy has metasubject nature, which is confirmed, first of all, by the concept of PISA: cognitive skills are stated as the most important characteristics by the study (Greene & Kellaghan, 2011). Moreover, to the three previously used characteristics– to formulate the problem mathematically, to apply mathematics and to interpret the mathematical result – by 2021 another one will be added – to reason, which has metacognitive nature (OECD, 2016).

The conclusion is obvious: mathematical literacy cannot be reduced only to the subject matter. Subject knowledge, no matter how practice-oriented it can be, for their successful application requires also the possession of skills related to work with information, semantic reading, speech skills and social awareness (Kautz, Heckman, Diris, Weel, & Borghans, 2014). Consequently, mathematical literacy, in addition to the subject component, includes a metasubject component associated with communicative, informational, reading and social competencies (Vinogradova, 2018).

In this regard, the question of appropriate didactic equipment is natural in the education. Currently, the majority of textbooks are not prepared to address this task, because it requires special methodological instruments and learning task (Li & Kaiser, 2011). Just as the developed project tasks are required to organize student project activities, so to form functional literacy it is required to use special story tasks based on real situations from the life.

Some researchers analyze text problems if they can be used in teaching the mathematical knowledge usage in life situations (Larina, 2018). Text problems have their own typology: tasks for movement, for relations, for parts, etc., each type corresponds to a certain mathematical model. However, we must understand that these didactic instruments which were developed for centuries, are not directed to the external world in relation to mathematics. On the contrary, it is intended to serve the mastery of mathematical content and the development of various mathematical models. And the application in this case is either absent or even contradicts the idea of mathematical modeling. Therefore, just as the element of educational content are text tasks, the same item of content should be tasks from the real life.

The next aspect highlighted by researchers in this area is the stories. All of them agree that mathematical literacy cannot be reduced to solve only everyday problems (Jablonka, 2003), that the social aspect in general has an independent significance for the education content (Suvorova, Roslova, Kuznetsova, & Minaeva, 2010). We should not forget that only in modern Russia it became possible to implement elements of probability theory and statistics into the content of mathematical education. It is impossible to form probabilistic thinking, statistical culture, critical thinking, if there is no place for variability in public life, opportunities for critical evaluation of aspects of the surrounding reality. This is directly related to the life of society, its values and attitudes.

In one of the discussed projects of Federal state educational standard of secondary education (FSES SE) it was offered to insert such didactic units as «hot water counters» and similar to activate the format of mathematical literacy. We will not discuss the legality of «diluting» of mathematical concepts, ideas and procedures by the terms that are far from mathematics. We will just speak out doubts that hot

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water counters will cope with the task to form mathematical literacy. And we will refer here to the fact that in the PISA study one of the characteristics of the task is the context of the situation, there are four: in addition to personal life, there are education/professional activity, social life, science.

Another aspect of the problem under discussion is the lack of systematic interdisciplinary communication at the program level.

Here is one more example of intersubject interaction problem. In the TV show-competition «We are literate» for students of 7-11 grades one of the regular stages is competition for the ability to persuade numerals. Generally speaking, the ability to incline numerals and correct use of numerals in speech is an element of the program in "Russian language" subject, numerals are invisibly present in the "List of elements of content tested in the main state exam in the Russian language" (Codifier of elements of the content and requirements to the level of training of students for the main state exam in the Russian language, 2018). However, there are almost no 100% results in this task for the participants of the competition.

Speaking about interdisciplinary interaction in the documents it should be noted that there is no mention of individual parts of speech in the Exemplary general educational program of secondary education (EGEP SE), there is only the "traditional classification of parts of speech", in the EGEP of primary education numerals are not mentioned in the subject "Russian language", however, in the section "Graduate will learn" in the subject "Foreign language" it is written: "recognize in the text and use in speech the studied parts of speech: ... quantitative (up to 100) and ordinal (up to 30) numerals; …" (Exemplary general educational program of primary Secondary education, 2015, p. 45).

Let us consider the documents that regulate the content of Russian mathematical education in the 21st century according to the indicated positions.

The emphasis on practice-oriented tasks was made in 2004, when the Federal component of the state educational standards of primary, basic and secondary (complete) education was adopted (Federal component of the state educational standards of primary General, basic General and secondary (complete) Secondary education, 2004). In the target section it was recorded that "the study of mathematics at the stage of basic Secondary education is aimed to achieve the following goals: mastering the system of mathematical knowledge and skills necessary for application in practice, the study of related disciplines, continuing education." This provision is detailed further in the requirements for the level of school graduates. The section "know / understand" describes the conceptual and conceptual aspects that provide application. E.g.,

"how mathematical formulas, equations and inequalities are used; examples of their application for solving practical problems;

how mathematically defined function can describe the real relations; examples of such descriptions;

probabilistic nature of many laws of the surrounding world;

examples of geometric objects and statements about them that are important for practice;

the meaning of idealization, allowing to solve real-life problems with the help of mathematical methods, examples of errors arising from idealization."

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A special section "The use the obtained knowledge and skills in practice and everyday life" provides specific requirements for each line of the mathematics course. E.g., in the part of the algebraic component it is revealed as follows: "use ... to:

* perform calculations with formulas, drawing up formulas expressing the relationship between the real values;

* modeling practical situations and research of the constructed models using the instruments of algebra;

* graphs interpretation of real dependencies between values".

It can be noted that this approach is consistent with the position of researchers who divide mathematical knowledge into conceptual and procedural (Rittle-Johnson & Schneider, 2014).

One more factor should be noted: in 2004, in the content of mathematical education of the basic school there was included a new content line – elements of combinatorics, statistics and probability theory, which has a wide practical application in the modern world.

These ideas have been developed in the framework of updating the state final certification of secondary school graduates - introduced in 2005 exam in mathematics in a new form (Kuznetsova, Suvorova, & Roslova, 2006). In accordance with the developed codifier of content elements in the examination papers there were included tasks aimed to check the ability to apply knowledge in real life contexts, and since 2010 tasks on probability theory and statistics.

The next stage was the introduction of the Federal state educational standard of Secondary education, which introduced metasubject learning outcomes, aimed, in particular, to develop the ability to apply the knowledge in practical situations. However, the contribution of individual subjects to form the claimed metasubject results was not recorded, that significantly hampered their implementation in educational activities. In addition, the line of practical use of subject knowledge has not found its development neither in the Federal state educational standard of basic Secondary education (2009), nor in the Exemplary general educational program (Exemplary general educational program of basic Secondary education, 2015). There was no place in this document for the fundamental core, which includes the functional component of the training of graduates, as well as to reflect interdisciplinary connections, despite their stated entry as a component in the metasubject learning outcomes.

At the same time, in order to preserve the continuity, as well as the implementation of the ideas of the FSES and accelerate their implementation in the practice of teaching mathematics, in the structure of the examination work in mathematics of the main state exam there was allocated a section "Real mathematics". It includes problems of practice-oriented orientation in arithmetic, algebra, geometry, statistics and probability theory. This step attracted some attention of teachers to the tasks encountered in life, however, did not solve the problem as a whole (Demonstration version of the control measuring materials for the main state exam in mathematics in 2019, 2018).

However, coordinated work on the standardization of education and the progressive development of the relevant system of state final certification could not affect the quality of mathematical education, in particular, the results recorded by the international study PISA. Thus, the total score of Russian participants increased from 468 points in 2009 to 494 points in 2015 (Results of the international study PISA 2015, 2016). In 2015, the average results for OECD countries for each type of cognitive activity

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and for each area of content increased, but one-Uncertainty and data, however, it can be stated that a significant gap due to the absence of these issues in the Russian program until 2004 has been eliminated.

It should be noted that the changes that we see are the result of standardization in 2004, because students who started to study under the new FSES from primary school, have not yet reached the participation in PISA – in 2019 they are finishing the 8th grade. So far, we can talk about the impact of general trends in the development of education rather than its systemic changes.

In this regard, there is a question about the mechanism that can ensure the form of mathematical literacy. However, at present there is no answer to a more general question: the mechanism for updating the content of Secondary education has not been worked out, moreover, the problem of the main document defining this content and, as a consequence, the content of the State final attestation (SFA) has not been solved.

Due to the fact that there is no answer to the question of how the content of education should be presented in the FSES: whether only through the requirements for the results of training or with the addition of a fundamental core, a mandatory minimum of content, etc. structural components, traditionally presented in the form of didactic units, there are various projects that have not yet found their final decision in the form of a legal document. However, it can be assumed that the expediency of presenting the content in the form of didactic units will eventually be recognized. In accordance with it, one may wonder how the considered real-life challenges should be reflected in this paper.

It was shown above that we had a positive experience: this is a two-level description of the educational content in the form of didactic units required for the provision of educational organization, and the requirements for the results of education required for graduates. The mandatory minimum content of education in 2004 did not include questions reflecting the aspect of practical application, but were sufficiently structured and detailed in the system of requirements.

Through the didactic unit there were described by two main components: “concepts” and “activities and operations”. It is advisable to allocate a separate component of “logic” and expand its content s to ensure the progressive development of skills of logical reasoning, substantiation, the ability to draw conclusions, formulate decisions. It is necessary to add two new components related to the application of mathematics. This is “mathematics and the outside world”, which doesn’t include stories from the life of the layman, but includes mathematical modeling of real-world situations, the use of mathematical instruments for solving problems and interpreting the meaning of the mathematical results in the real situation. It is “mathematics in the system of science and technology”, which is responsible for the form of interdisciplinary connections and experience in the application mathematics during the learning other school subjects.

You can refine and specify the requirements for learning outcomes: move from general to more specific. E.g.: to be able to readout the units (household, scientific), to use scales, to evaluate and approximate the results of measurements and calculations in the arithmetic section, to measure subjects and objects, to analyze structures in geometry, to understand the probabilistic nature and to assess the chances of events of social reality and phenomena of the world: natural, weather – in the probabilistic and statistical section.

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Separately, you can formulate requirements that reflect interdisciplinary interaction, in particular, the use of mathematical knowledge in the study of other subjects or, conversely, the use of subjects from other subjects, descripting them in the mathematical language.

It is advisable to continue work on the control measuring materials. The category of “Real mathematics” is removed, because each task is again received substantive reference to algebra or to geometry. At the same time, the tasks of probabilistic and statistical content were “in algebra”, while the exam is called “mathematics”. Again, there was a certain contradiction and it was illogical.

Let's look at the contexts of the six tasks of the demo version of 2019: reading the graph, the diagram about dried mushrooms, the geometry of the barn roof, working with the formula for transferring temperatures from the Celsius scale to the Fahrenheit scale all of that has a scientific context; calculating the cost of travel, determining the probability in the problem of pies on a plate – it is personal life (Demo version of the control measuring materials for the main state exam in mathematics in 2019, 2018). It is easy to see that the tasks are dominated by scientific contexts. Actually, there are no plots, rather, again there is a didactic mathematical problem with a hint of the context of a particular branch of knowledge.

7. Conclusion

The use of digital technologies has made it possible to conduct large-scale monitoring studies covering various educational systems of the world, as well as the use their results to analyze the state of the education system and find ways to improve the quality of training of school graduates. Currently, international research is developing in two directions: providing access to participation in projects not only for States, but also for individual regional systems, as well as computerization of research technology. All this will allow eventually to get more and more information and individualize the results. Now it is necessary to learn to analyze the results and conclusions of research in the context of the changes occurring in the education system, use them in the interests of developing and improving the quality of Russian education, improving the regulatory framework and working with teachers (Greene & Kellaghan, 2011).

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