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MOTIVATION IN TEACHING MATHEMATICS

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Abstract

The relevance of the article is conditioned by the fact that motivation is one of the major prerequisites for the improvement of the quality of teaching mathematics.

Motivation in studying, as a type of motivation, depends on a number of factors: the educational system, organisation of the educational process, the individual features of the educator and the student. This paper touches upon such an insufficiently investigated topic as motivation in teaching mathematics.

Research and pedagogical experience indicates that both students and teachers who are just starting their career underestimate the role of motivation in teaching mathematics, and their skills in motivating are insufficient.

The aim of this research is to identify the abilities of mathematics to motivate and the methods of training students to use those abilities during their teaching careers.

The research methods used in this study are theoretical analysis of the current state of the problem, questioning, and participant observation.
This research has showed that students and teachers who are just starting their career know basic methods of motivating but do not recognize the motivational potential of mathematics and the exact topics with which this potential can be realised. In addition, it has been determined that motivational work can be implemented while introducing mathematical terms and algorithms, learning new operations, doing exercises, and proving theorems.

1. Introduction

Motivation is one of the major prerequisites for the improvement of the quality of teaching mathematics. Motivation has a great influence on the mental state of the students, their satisfaction from learning, and their ability to make different decisions, including those related to studying (Zimnyaya, 2005).

The psychological basis for motivation is discussed in detail in studies of such scholars as Leontyev (1971), Maslow (1970), Ananyev (2002), Bozhovich (1997) and Lewin (2001). In their studies, motives are viewed as persistent manifestations of the human personality that stimulate the person to act, while motivation is defined as the sum of all inciting factors that characterise actions of the individual (Ananyev, 2002; Bozhovich, 1997; Maslow, 1970; Lewin, 2001).

Pedagogical aspects of motivation are considered in works of such researchers as Shchukina (2006), Markova (1990). They determined the pedagogical conditions for the activation of the pupil's motivation: selection of such content for the course that will encourage the students to take their own educational paths, an individualised approach to the design of the educational process, selection of teaching methods on every stage of the educational process, and the type of interaction between the participants of the educational process (Shchukina, 2006; Markova, 1990).

2. Problem Statement

Motivation in studying, as a type of motivation, depends on a number of factors: the educational system, organisation of the educational process, the individual features of the educator and the student, etc. This paper deals with such an insufficiently investigated topic as motivation in teaching mathematics.

3. Research Questions

Research and pedagogical experience indicate that both students and teachers who are just starting their career underestimate the role of motivation in teaching mathematics, and their skills in motivating are insufficient. They omit motivational work intentionally or unintentionally during the introduction of mathematical terms, while studying theorems, learning new operations, and discussing approaches to doing exercises. In other words, they do not see the potential of everyday class activities to motivate.

4. Purpose of the Study

The aim of the research is to identify the abilities of mathematics to motivate and the methods of training students to use those abilities in their teaching careers.

5. Research Methods

The research methods used in the study are theoretical analysis of the current state of the problem, questioning, participant observation, and analysis of the results of creative work. The participants of the research are students and graduates (who are starting a career in teaching) of the department of pedagogical education at Lobachevsky Institute of Mathematics and Mechanics and course participants at Volga Region Centre for Advanced Training and Professional Retraining at Kazan Federal University.

While covering methods of teaching mathematics with our students, we discuss motivation techniques which can be applied by future mathematics teachers:

- teachers should motivate their pupils by personal example. It must be noted that not only professional but also personal qualities of the teacher play a role in this process. It is important both to show the importance of mathematics for the functioning of society and to be friendly, opportunely address the questions of the students, be able to hear out their opinions, and give the students an opportunity to express their position fully even if their conclusions are based on obviously wrong or inaccurate assumptions which can sometimes deviate from the topic discussed in class;

- teachers must be considerate towards the students' opinions on relevant topics and handle occurring difficulties in cooperation with the students. The teacher's ability to support the students, the ability to boost the students' confidence in their own skills with an encouraging word, and the ability to emphasise the importance of the students' deliberations with as little as a simple facial expression will help to organise educational process correctly and establish a proper relationship with the students;

- demonstration of how to apply mathematical knowledge in practice is also of great importance. Multidisciplinary classes that combine mathematics and economics, mathematics and physics, and mathematics and geography allow the students to see how these disciplines are intertwined in actual everyday practice and discover cognitive motives. Through the use of such combined classes, the students start to value the end result of the education more than an intermediate grade;

- teachers should distinguish the students' achievements, commend them for valid assumptions, and encourage unconventional approaches to problems. Such work yields immediate results: the students gain greater confidence in their abilities and find inner motivation for revising and improving previous results. Another important aspect is that the students must know how classwork, tests, and homework are assessed. Transparency and objectivity of the assessment criteria are effective means of motivation (Timerbayeva & Fazleeva, 2017);

- teachers should alternate different forms of classwork. Since attention can be held only for 15-20 minutes in average, teachers must employ the whole range of educational activities: oral tasks develop memory and the ability to focus; a graded system of exercises allows the teacher to compare the students' mastery of the course with the difficulty of the given exercises; discussion of the topic at hand or a case study provides an opportunity to express an opinion, defend one's position, and express the logic of mathematical operations correctly; creative work demands the use of special and interdisciplinary knowledge; the use of historical references and illustrations stimulates cognitive interest; etc. (Timerbayeva, Fazleeva, & Shakirova, 2017);

- students must be given manageable tasks and fully understand what their homework is. The difficulty of the given tasks must be only slightly above the actual proficiency of the students. It is important to encourage the students to search for lacking information on their own. At the same time, teachers must analyse the results of the submitted exercises and help the students to find the right solution. Clear formulation of the homework and explanation of its role also contribute to the students' motivation (Fazleeva & Timerbayeva, 2017);

- students must be tasked to check and assess written and oral work of their classmates. Teachers' trust in their students acts as another means of developing motivation, while the desire to make an objective assessment helps the students to stay focused in class;

- teachers must encourage those students who perform their tasks fast. For this purpose, educators use a wide range of cards with additional tasks of varied difficulty, which motivates the students to work hard for a high grade. Those students who are prone to discussion should be offered speech topics for the following classes;

- teachers must encourage the students to participate in conferences and academic competitions in mathematics.

The professional standard of the teacher outlines the competencies that will improve students' motivation for learning. The teacher must:

A) create an environment in which the students will gain satisfaction from studying mathematics;

B) help the students to understand that knowledge of mathematics will be useful regardless of their future profession;

C) take part in the students' preparation for research projects, conferences, and academic competitions in mathematics;

D) be able to recognise and maintain high motivation, give workshops, optional and elective courses;

E) ensure that at least one of the exercises that a student does in class or as part of homework is assessed and the result is said to the student (Professional standard "Teacher", 2014).

The aforementioned competencies were assessed on a scale from 0 to 2 in a group of students and teachers with different teaching experience, and the results are given in Table 1.

Table 01. Competency assessment

Assessed competencies	Bachelor's programme students (4th-5th years)	Teachers (teaching experience of 1-3 years)	Teachers (teaching experience of 5 or more years)
A	1	1	2
B	0	1	2
C	0	0	2
D	0	0	1
E	0	1	2

Observations showed that the difficulties which young teachers of mathematics encounter are born from a number of factors: their inability to organise mathematics classes in such a way as to cause positive emotions in the students, inability to show the importance of mathematics for everyday life, inability to administer workshops and elective courses, and inability to train students for participation in mathematical competitions. Another notable factor is the teachers' inability to organise classes in such a way as to provide every student with an opportunity to do at least one exercise correctly in class or at home. It also must be noted that even the standard of the teacher itself does not clarify the question of motivating students through the discipline they study.

In this context, we decided to touch upon the question of whether it is possible to motivate students at mathematics classes. At a class dedicated to methods of teaching mathematics, third-year university students (28 people) were asked a question: how would you motivate pupils while discussing the topic of scale in sixth grade?

The students' answers are as follows:

- show how this concept is used (3 students);

- start by drawing a comparison between two maps (3 students);

- start by relying on the pupil's previous experience and ask them how they understand the word "scale" (3 students);

- offer to do a relevant exercise (6 students);

- start directly by announcing the topic (13 students).

To summarise, a significant part of the students (46%) suggested explaining this topic to pupils starting from abstractions and deduction. A small number of students (21%) suggested starting by giving a relevant exercise; however, they could not give an example of an exercise that would help them to introduce the new concept.

In order to determine the place that people give to motivation in teaching mathematics, we have questioned first- to fifth-year university students (181 people) and working educators with different teaching experience (61 people). The participants were asked to rank a number of factors from the most to the least important.

Table 02. Factors that improve the quality of teaching mathematics

Rank the factors that improve the quality of teaching mathematics
____ Timely control   ____ Motivation   ____ The classroom's equipment   ____ Discipline in class   ____ The student's willingness to study   ____ Individualised approach to every student   ____ Techniques of education through play   ____ The student's interest in the subject   Other   factors   _____   _____   _____   Give your teaching experience   year

6. Findings

We have summarised those forms in which the survey participants gave the motivational factor first, second or third position. The results of the questioning are shown in Table 3.

Table 03. Importance of motivation

Rank	Students (% to the total number)					Educators
	1st year	*2nd year*	*3rd year*	*4th year*	*5th year*	
1	14.3	7.8	20.0	33.3	23.3	24.6
2	14.3	11.5	25.0	17.9	14.0	31.1
3	25.7	19.2	25.0	17.9	14.0	18.0
4 or lower	45.7	61.5	30.0	30.9	51.7	26.3

Table 3 shows that the importance of the factors affecting the process of teaching mathematics fluctuates depending on the year of the students' education. It is also evident that motivation is ranked as one of the most important factors (in each case it was given one of the first three positions). Speaking of the questioned teachers, one-third have ranked motivation as the second most important factor (which is in

compliance with the logic of methodology), and one-fourth have given motivation first position. Therefore over a half of the teachers have given motivation a leading role.

Within the framework of the research, it was decided to assess the classes administered by students during their internship and teachers who are only starting their career. Specifically, reports and self-assessments forms submitted by the aforementioned students and teachers, video recordings of the administered classes, and the results of tests written by the pupils were analysed.

Assessment revealed that students in their internship and young teachers are unable to motivate properly, and, more specifically, they cannot use the potential of mathematics for motivation. Some examples of proper motivational work at mathematics classes are given below.

In sixth grade, the teacher starts a mathematics class by stating the scheduled topic ("The linear function") and gives the definition without any preparatory activities with the pupils. It is important to note that instead of a simple statement of the definition, the pupils may be offered to deduce the definition themselves. For example, once the teacher has drawn a series of points belonging to one function, the pupils may realise on their own that the points form a straight line; hence the name of the function. Therefore, in this example, the teacher fails to use an opportunity to motivate the pupils to define the new concept.

At a class dedicated to operations with fractions in fifth grade, the teacher writes equation $\frac{1}{3} \cdot x = 12$ and tells the pupils to multiply both sides by 3. However, the teacher does not emphasise the necessity of turning the multiplier of the unknown quantity into an integer: $3 \cdot \frac{1}{3} = 1$. Thus, in this example, the teacher fails to lead the pupils to a new operation.

While covering the topic of prime and composite numbers, the teacher demonstrates the application of the sieve of Eratosthenes but does not lead the pupils to the notion of sieve.

If a teacher needs to lead the pupils to the idea that zeroes after a decimal mark can be omitted, then he or she must render the number as a usual fraction in which the last zeroes can be cancelled.

In this research, we emphasise the skill in using the immediate content of a mathematics course to motivate the pupils. Teachers cannot or do not consider it necessary to lead the pupils to new concepts, mathematical operations, or approaches to solving problems. Entire classes must be allocated to the development of such skills in future teachers.

During classes dedicated to methods of teaching mathematics, we show our students how motivation can be combined with certain elements of mathematics.

1) Introduction of new concepts.

- Introduction of the concept of logarithm in eleventh grade. A class is started with solving an equation: $2^x = 8$. In order to solve it, the equation must be modified in a way to make both sides have the same base (2). Then the pupils are asked to solve another equation: $3^x = 5$. The same approach cannot be applied here. However, the equation does have a solution, which can be demonstrated graphically. In this situation, the teacher should tell the pupils that the unknown quantity represents the power to which 3 must be raised in order to obtain 5. Then the teacher introduces the concept at hand, thus leading the pupils to it.

- Introduction of the concept of scale in sixth grade may be started with an exercise. The teacher asks the pupils to calculate the distance (for example, between cities) with the use of a map.

• Introduction of a new representation of the quotient of a and b ($a:b$) as a fraction $(\frac{a}{b})$ can also be started with an exercise. Such an approach will bring about the necessity and an opportunity to represent the quotient in a new way. The pupils will notice that these two symbols are shown combined on the respective calculator key ($\div$).

• Introduction of the concepts of sine and cosine of an acute angle must take place after the discussion of similar right-angled triangles. The teacher points out that the ratio between the sides of similar triangles (for example, the ratio of the opposite cathetus to the hypotenuse) is constant, thus it is logical to give this ratio a certain name ("sine")

2) Doing geometry exercises.

The pupils are tasked to calculate the distance from a given point to a given plane.

To solve this task, a perpendicular must be drawn from the point to the plane. There are a number of ways to perform that operation. For example, one can find two concurrent straight lines on the given plane and draw a perpendicular towards them. Another approach is to draw a perpendicular towards the line of intersection of two mutually perpendicular planes, where one of the two planes is the one initially designated in the exercise, and the other plane includes the designated point. Still another option is to draw a random perpendicular to the plane and then draw another line which is parallel to the first and goes through the designated point. Knowledge of these options (or their revision prior to giving the exercise) may motivate the pupils to deduce the solution.

Examples of different approaches to calculating the distance between a point and a plane are given below.
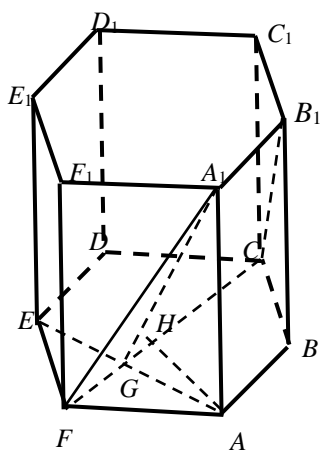
Exercise 1. In a regular hexagonal prism $ABCDEFA_1B_1C_1D_1E_1F_1$ whose edges equal 1, calculate the distance from point A to plane A_1B_1C.

Solution. Plane AA_1E is perpendicular to plane A_1B_1C which contains straight line FC, and these two planes intersect along line A_1G. Therefore the length of altitude AH in triangle AA_1G is the solution to the exercise.

In right-angled triangle ADE we calculate

$AE = \sqrt{AD^2 - ED^2} = \sqrt{3}$, consequently $AG = \frac{\sqrt{3}}{2}$.

Then in right-angled triangle AGA_1 we calculate

$GA_1 = \sqrt{\frac{3}{4} + 1} = \frac{\sqrt{7}}{2}$, and the end result is $AH = \frac{AG \cdot AA_1}{GA_1} = \sqrt{\frac{3}{7}}$.

The answer: $\sqrt{\frac{3}{7}}$.

This exercise is completed with the use of the second approach. In this case, the teacher says only "which" operations to use but does not explain "why". The solution itself remains "concealed" to the pupils. Until the very last step, the pupils do not know why they are performing these operations: draw a plane that

goes through point *A* and is perpendicular to plane A_1B_1C; find the line of intersection between the drawn plane and the designated one; etc. Basically, none of the operations is justified to the pupils. Only at the end of the solution does the logic become clear, while the most important aspect of any solution is to answer the question "where do I start?"
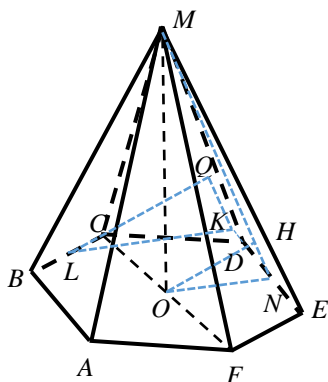
Exercise 2.

In a regular hexagonal pyramid *MABCDEF* in which the side edges equal 4 and the edges of the base equal 1, calculate the distance from the middle of edge *BC* to face *EMD*.

Solution. In a regular hexagon $ABCDEF, BD \perp DE$ and $BD = \sqrt{3}$. Since *MO* is the altitude of the pyramid, then in right-angled triangle *MOD* we find $MO = \sqrt{15}$. In right-angled triangle *MDN* we find the apothem of face *DME:*

$$MN = \sqrt{MD^2 - \left(\frac{DE}{2}\right)^2} = \frac{3\sqrt{7}}{2}.$$

Planes *MON* and *DME* are perpendicular; therefore altitude *OH* of triangle *MON* is perpendicular to plane *DME*.

In right-angled triangle *MON* we find

$$OH = \frac{MO \cdot ON}{MN} = \sqrt{15} \cdot \frac{\sqrt{3}}{2} : \frac{3\sqrt{7}}{2} = \sqrt{\frac{5}{7}}.$$

From point *L* and the middle of *BC* we draw perpendicular *LQ* towards face *DME* and perpendicular *LK* towards line *DE*, which gives us triangle *LQK* that is similar to triangle *OHN*. Their similarity ratio equals $\frac{3}{2}$.

Then $LQ = \frac{3}{2} \cdot OH = \frac{3}{2} \cdot \sqrt{\frac{5}{7}} = \sqrt{\frac{45}{28}}$. *The answer*: $\sqrt{\frac{45}{28}}$.

This exercise is completed in accordance with the third approach. As before, the teacher explains only the succession of operations, while the most important task, finding (or drawing) a line that is perpendicular to plane EMD and parallel to the perpendicular whose length will be the answer to the exercise, remains unknown to the pupils until the very last moment.
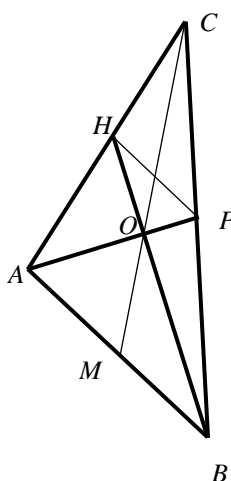
Exercise 3. Two medians of a triangle equal 9 and 12 and are perpendicular to each other. Calculate the length of the third median.

Solution. We start drawing the triangle by drawing two mutually perpendicular straight lines. Then, towards the opposite sides from the intersection point *O*, we mark segments whose lengths satisfy a ratio of 2 to 1. Next we complete triangle *ABC*.

The next step is to prove that *BH* and *AP* are the triangle's medians. Then we apply the properties of a triangle's medians and find the lengths of segments $AO = \frac{2}{3}AP = 6$ and $BO = \frac{2}{3}BH = 8$.

In right-angled triangle AOB, we find the length of hypotenuse AB:

$$AB = \sqrt{AO^2 + BO^2} = 10.$$

Since OM is a median of this triangle and is drawn from the vertex of a right angle, then $OM = \frac{1}{2} AB = 5$. After that, we calculate the length of the necessary median: $CM = 3 \cdot MO = 3 \cdot 5 = 15$.

The answer: 15.

Exercise 3 was given to a group of 28 students in their third year at a university. 11 people completed the exercised correctly; 1 student has found the solution by drawing a figure while preserving the designated proportions. 16 people did not complete the exercise correctly. It would have been possible to lead the students to the right solution by revising the properties of a triangle's medians and the properties of a right-angled triangle's medians prior to giving the task. Another viable option was to start by drawing a figure in accordance with the designated parameters of the exercise, that is, by drawing two perpendicular lines and marking segments from the intersection point of the lines with the ratio of the segments' lengths 2 to 1.

It is evident that the choice of the approach to completing the task is closely related to the necessity of performing analysis. Otherwise, the pupils will have to act through trial and error.

7. Conclusion

This research shows that students and young teachers know basic methods of motivating but do not use the motivating potential of mathematics as a subject and do not see how exact topics can be used to realise this potential. In addition, teachers often offer unmotivated solutions to the exercises without leading the pupils to the right answer, which is necessary to avoid because pupils must learn the general approaches to completing an entire group of typical tasks instead of simply knowing operations required to complete a concrete task.

It has been determined that motivational potential of mathematics as a subject can be realised while introducing mathematical terms and algorithms, learning new operations, doing exercises, and proving theorems.

Improvement of the training of future mathematics teachers should be directed towards the development of mathematical, pedagogical and methodological thinking in students, which is a necessary prerequisite for implementing motivation at mathematics classes. In order to perform that task, classes in methodology of teaching should emphasise the acquisition of skills in motivation that will be applicable over the entire school course of mathematics. Moreover, the existing training programmes for mathematics teachers should be supplemented with the following elective courses: "Motivation techniques," "The role of motivation in teaching mathematics," "Motivation for learning through the content of the subject," and others.

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