

**RPTSS 2017**  
**International Conference on Research Paradigms Transformation  
in Social Sciences**

**FORMING NUMERICAL MODEL FOR CALCULATING  
OPTIMAL TAX RATE FOR RESOLVING STAKEHOLDERS'  
INTERESTS**

K.A. Bannova (a)\*, N.E. Aktaev (b)

\*Corresponding author

(a) National Research Tomsk Polytechnic University, Lenina Avenue, 30, Tomsk, Russia, E-mail:  
bannovaka@yandex.ru, +79528867402

(b) National Research Tomsk Polytechnic University, Lenina Avenue, 30, Tomsk, Russia, E-mail: aktaevne@tpu.ru,  
8 (3822) 60-64-05

*Abstract*

For a long time by means of taxes, many state issues of redistribution of finance were resolved, and its size defined the standard of living of the citizens residing on a certain territory. By means of taxes, it is possible to increase the standard of living of population, to provide competitiveness of companies within the country. Competitiveness of a company is defined by many factors, such as the level of expenses, including payment of taxes, which is key. For lack of the competition in the domestic market, the tax component does not strongly influence the consumption level, but determines production price. However, when entering the world market or when competing in domestic market with foreign goods, the lowering of the level of expenses is a necessary condition not only for the successful competition, but also for existence of the company as a structural economic unit. However, it is necessary to know the optimal level of tax rate to provide low cost, on the one hand. But, on the other hand, there is a positive dynamics of the state budget. The authors assume that at the optimal level of the tax rate, the value that is the ratio of the company's key indicators is maintained. The dynamics of the company's development is determined by keeping the ratios between the indicators as a constant value. It is assumed that these ratios will serve as an indicator of competitiveness and will determine the company's development strategy in the domestic and foreign markets.

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**Keywords:** Consumption, tax rate, tax, history of taxes, optimum tax rate



## 1. Introduction

History of the taxation contains several millennia. The first taxes had an unsystematic character and were raised as required (wars, major public projects) in a natural form (food for officials and army, construction materials, labor) (Smith, 1922). In the process of the increase of expenses of the state taxes have begun to be raised in cash more regularly and became the main source of monetary pay of the state. So, long time in economy fundamental was classical idea of taxes (Harrod, 1951). More than 200 years ago, A. Smith has for the first time created the doctrine about evolution of public finances in which it was claimed that formation of the tax system happens at a rather high level of development of the state (Friedman, 1997). The tax system, by A. Smith's definition, is more or less ordered and systematic withdrawal of a part of the income of independently economic entities. He has put forward four principles of the taxation: uniformity, definiteness, convenience of payment and low cost. Scientific classics of U. Petty, Zh.-B. Sei and English economists of D. Ricardo, J. Mill considered economy as a steady and self-regulating system in which demand gives rise to the offer, and at a surplus of any of them, there is a self-equilibration by movement of producers in scarce branches of economy (Raico, 2008). Taxes played only a role of sources of the income of the budget of the state, and polemic was conducted around the principles of justice of their collection (uniform or progressive) and parts of the withdrawal caused by fiscal requirement (Rothbard, 1992). But with complication of the economic relations in society, under the influence of objective cyclic processes there was a need for updating of the classical doctrine, allocation of the influence of the state on economic processes, and also regulation of social and economic borders of the taxation. There were new scientific theories. Now it is possible to allocate two main directions of an economic thought - Keynesian and neoclassical.

Keynes & Weber (1998) was a supporter of progressiveness in the taxation as considered that the progressive tax system stimulates adoption of risk by the producer of rather capital investments. An important condition of the theory of Keynes (2004) is dependence of the economic growth on sufficient monetary savings only in the conditions of full employment. If there is no it, then big savings hamper the economic growth as represent a passive source of the income, don't invest in production. Therefore, in his opinion, excessive savings should be withdrawn by means of taxes. And for this purpose the state intervention aimed at withdrawal redistribution by means of taxes of the income displaced in savings, and financing at the expense of these means of investments, and also the operating public expenditures is necessary.

According to Keynes's theory, taxes work in the economic system as "the built-in mechanisms of flexibility". High (progressive) taxes, in his opinion, play a positive role. Being an integral part of the budget, this economic category influences balance in economy. So, a decrease in tax revenues reduces the income of the budget and aggravates economic instability. And taxes as "the built-in stabilizer" smooth this process: during economic recovery, the assessed income grows more slowly than the tax income, and at crisis the tax decreases quicker than the income falls. Thereby a rather stable social status in society is reached. It is obvious that tax revenues in the budget depend on the size of a tax rate and on the size of the income assessed on an ascending scale of income taxation. However, fluctuations are more considerable in tax revenues than in the level of the income. Keynes's theory was successfully put into practice by the

English government before the World War I when all forces of the state have been aimed at the development of military industrial production.

It is considered that under conditions of a scientific and technical rise, more and more frequent manifestation of the crisis phenomena the Keynesian theory of intervention of the state through achievement of "effective demand" has ceased to meet the requirements of economic development (Crist, Laffer, 1982).

## **2. Problem Statement**

Almost all modern writers (Dolgih, Zhdanova, Bannova, 2014) mark out the same criteria of taxes estimating their influence. If taxes do not meet the requirements defining positive influence, then violation of borders of the taxation is available. A. Laffer (Laffer, Canto, Eastin, Kadlec, 1985; Laffer, 1977; Laffer, Zecher, 1975) has presented the quantitative dependence between progressiveness of the taxation and the income of the budget in the form of a parabolic curve – curve Laffera. This curve shows the character dependence of contributions to the budget on a tax rate.

Apparently from the analysis of the curve, the system of the taxation has a very rigid system of determination of the level of tax rates. At respect for the principles of the taxation of Laffer the state as if has no relation to the system of formation of tax rates. However, for many large countries, including Russia, the main part of tax revenues in the budget is provided by the large enterprises which activity is mainly focused on the foreign market. Therefore, the state is forced to support these companies from budget funds of the country, thereby providing stability of the economy. Thus, it is possible to assume that now the authors will use Laffer's mechanism in relation to domestic market and small enterprises.

The analysis of the modern literature shows that the optimum size of a tax rate is defined by an empirical way. At the heart of empirical approaches to management of financial streams taxes are a part of investments and/or potrebleniye. In this case, there is no opportunity to consider separately the tax component and, as a result, her influence on financial streams of the company. In the real work, the model of calculation of an optimum trajectory of development at identification of an optimum tax rate is offered.

## **3. Research Questions**

The analysis of communication between the economic growth and taxes over 20 countries during the 1970th demonstrates that in the states with high taxes, such as Ireland, Venezuela, Japan, Austria, Aruba, Belgium, the Netherlands of investment have decreased in comparison with the same indicator in the countries with the high level of the taxation almost ten times: Great Britain, Sweden, the USA, Denmark, Finland, etc. Widespread practice in the analysis of an optimum share of withdrawal have arrived to the budget, the comprehensive empirical investigation of activity of the enterprises in case of application of other interest rates is. However, their level should not be minimum, otherwise requirements of formation of the budget will not be observed and normal functioning of public economy is threatened, and taxes will cease to perform functions of the economic regulator.

Thus, the theory of economy of the offer developed in the USA and Great Britain by M. Ueysdnbaum, M. Boris, G. Stein, A. Laffer, were guided by the main idea consisting in a radical

reduction of limit tax rates and decrease by that, progressiveness of the taxation. It has led to the fact that the enterprises would increase their own sources of accumulation.

#### 4. Purpose of the Study

Model for calculating the optimal tax rate

#### 5. Research Methods

The methodology of our research is based on: production function, principle of a maximum Pontryagin, Hamilton function, parameter of discounting. Theoretical and methodological basis of a research is use of the principles and methods of scientific knowledge, use of provisions of the theory and a technique of the taxation. In a research of problems on the chosen subject authors lean on fundamental and special literature of domestic and foreign authors, legislative and regulations, publications on a research perspective on thematic pages of the Internet. In the course of the research such general scientific methods of knowledge as the analysis and synthesis, systemacity and complexity, abstraction, comparison and logical generalization will be used.

Proceeding from it, according to classical representations, temporary dependence of production  $Y(t)$  is defined by consumption  $C(t)$  and investments  $I(t)$ .

$$Y(t) = C(t) + I(t) \quad (1)$$

In such statement taxes are considered in total with consumption and investments. We suggest to consider a tax component  $N(t)$  separately. Then

$$Y(t) = C(t) + I(t) + N(t) \quad (2)$$

Determining parameter  $S$  as a share on investment, and  $\chi$  as a share on a tax, we will rewrite expression (2)

$$Y(t) = [1 - \chi(t)] \{ [1 - s(t)] Y(t) + s(t) Y(t) \} + \chi(t) Y(t) \quad (3)$$

The main tool of description the production is production function  $F[K(t), L]$ , where  $K$  characterizes fixed assets,  $L$  – labour.

The main parameter of the use of production assets  $K(t)$  is depreciation, which characterizes the volume of the failed part of fixed assets at a rate  $\mu$ . Provided that the tax rate is influenced by the change in fixed assets, the change in the time of fixed assets is described by a differential equation.

$$\frac{dK}{dt} = s(t) [1 - \chi(t)] F[K(t), L] - \mu K(t) \quad (4)$$

For a quantitative assessment of influence tax rate of economic system we will write down functionality which needs to be maximized

$$\int_0^T C(t) L^{-1} \exp(-\delta t) dt \rightarrow \max \quad (5)$$

Where  $T$  – planning horizon,  $\delta$  – discounting parameter. Replacing consumption parameter with a production parameter, we will rewrite expression as

$$\int_0^T [1 - \chi(t)][1 - s(t)]Y(t)L^{-1} \exp(-\delta t) dt \rightarrow \max \quad (6)$$

Thus, we will formulate a problem of optimization in the general statement. It is necessary to maximize functionality

$$\int_0^T [1 - \chi(t)][1 - s(t)]F[K(t), L]L^{-1} \exp(-\delta t) dt \rightarrow \max \quad (7)$$

Provided that

$$\frac{dK}{dt} = s(t)[1 - \chi(t)]F[K(t), L] - \mu K(t) \quad (8)$$

and restriction  $0 \leq s(t) \leq 1$ . We will pass the decision of the specified problem of maximizing by means of the principle of a maximum Pontryagin. The essence of the solution lies in the search for optimal management  $s(t)$  and  $\chi(t)$ . The main tool for realization of the principle is Pontryagin's function or as still it is accepted to call, Hamilton function. For record of a Hamiltonian we will use communication, well-known from Hamilton mechanics, with Lagranzhian's  $\mathcal{L}$  (Aktaev, 2014; Pavlova, Aktaev, Gontchar, 2012).

$$H(K, \psi, t) = \psi \frac{dK}{dt} - \mathcal{L}(K, \psi, t) \quad (9)$$

Carrying an analogy with the apparatus of Hamiltonian mechanics in physics, we can interpret the value  $K$  as a generalized coordinate, and  $\psi$  dual variable - as a generalized impulse. These conditional definitions don't contradict common sense. Really, size  $K$  characterizes the coordinate of economic system, that is her fixed assets. Then,  $\psi$ , as it will be shown further, defines the movement of economic system in space of the generalized coordinates.

According to the principle of the smallest action integral in the ratio (7) defines lagranzhian with a negative sign (the principle of the smallest action determines the minimum value of integral by time from a lagranzhian). Then the Hamiltonian will correspond in a look

$$H(K, \psi, t) = \psi \frac{dK}{dt} + [1 - \chi(t)][1 - s(t)] \frac{Y(t)}{L} \exp(-\delta t) \quad (10)$$

Substituting expression (8) in (10), we will receive final expression for a Hamiltonian as

$$\begin{aligned}
 H(K, \psi, t) = \psi \{ & s(t)[1 - \chi(t)]F[K(t), L] - \mu K(t) \} + \\
 & + [1 - \chi(t)][1 - s(t)] \frac{F[K(t), L]}{L} \exp(-\delta t)
 \end{aligned} \quad (11)$$

We will find a condition under which Hamilton function accepts the maximum value. For this purpose it is necessary to differentiate it on an optimum trajectory  $s(t)$

$$\frac{\partial H}{\partial s} = \psi [1 - \chi(t)]F[K(t), L] - [1 - \chi(t)] \frac{F[K(t), L]}{L} \exp(-\delta t) \quad (12)$$

We will equate the received expression to zero. Then

$$\psi [1 - \chi(t)] F [K(t), L] - [1 - \chi(t)] \frac{F [K(t), L]}{L} \exp(-\delta t) = 0 \quad (13)$$

After some transformations we will receive

$$\psi(t) = L^{-1} \exp(-\delta t) \quad (14)$$

From expression (14) it is visible that the dual variable has the decreasing exponential dependence on time. Parameter of discounting determines the speed of reduction of a dual variable. In initial timepoint  $\psi(0) = L^{-1}$  also isn't defined by fixed assets.

On the other hand the dual variable is defined by the canonical equation of Hamilton

$$\frac{d\psi}{dt} = - \frac{\partial H}{\partial K} \quad (15)$$

Then, differentiating a Hamiltonian on the generalized coordinate  $K$ , we will receive

$$\begin{aligned} \frac{d\psi}{dt} = & -\psi(t) \left\{ s(t) [1 - \chi(t)] \frac{\partial}{\partial K} F [K(t), L] - \mu \right\} - \\ & - \frac{[1 - \chi(t)] [1 - s(t)]}{L} \frac{\partial}{\partial K} F [K(t), L] \exp(-\delta t) \end{aligned} \quad (16)$$

According to (14) the left part of expression (16) takes a form  $-\delta L^{-1} \exp(-\delta t)$ . Then

$$\begin{aligned} -\frac{\delta}{L} \exp(-\delta t) = & -\frac{\exp(-\delta t)}{L} \left\{ s(t) [1 - \chi(t)] \frac{\partial F [K(t), L]}{\partial K} - \mu \right\} - \\ & - \frac{[1 - \chi(t)] [1 - s(t)]}{L} \frac{\partial F [K(t), L]}{\partial K} \exp(-\delta t) \end{aligned} \quad (17)$$

After some transformations we receive

$$\chi(t) = 1 - (\mu + \delta) \left\{ \frac{\partial F [K(t), L]}{\partial K} \right\}^{-1} \quad (18)$$

Expression (18) characterizes optimum control of tax sector of regulation of activity of economic system. Feature of the received expression is universality in relation to the chosen model of production function.

## 6. Findings

Thus, we make numerical model for calculating the optimal tax rate for resolving the interests of state and taxpayers. The given model will allow to define an optimum trajectory of development of the companies focused on the foreign market, for maintenance of stability economy of state. At present, there are no approaches to universal mathematical modeling of tax. As a rule, existing models are of a private nature and allow an approximate numerical estimation of specific indicators for specific situations. The urgency of solvable problem is determined by need to find solutions to ensure the stabilization of the economy in the shortest possible time, with the systemic crisis of the world economy.

## 7. Conclusion

Availability of the theoretical model allows to simulate various types of economic situations and offer the most optimal way of reconstruction and development of the national economy. Scientific significance is determined using a theoretical study time dependences for estimating parameters. It should be noted that within the framework of the classical theory, as a rule time dependence was neglected and existing models are applicable in a stable economy and stationary processes. In the future, it is planned to assess the impact of the production function on the final equation of the model, as well as calculate the optimal share of taxes that the company should allocate for the development of its activities.

## Acknowledgments

The study was financially supported by grant of the President of Russian Federation for state support of young Russian scientists- Ph.D. in the frame of the project for scientific studies («Theoretical research of the taxation for large Russian companies in the conditions of instability market of commodity turnover for increase in competitiveness of national economy»), project No. MK-3443.2017.6.

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