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REALITY IN MATH: BARRIER OR HELP?

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Abstract

The scientific debate on factors of success in school problem solving is still running. One of the explored points is a connection between school problems and real-life problems. The study examines whether the experience with some real-life domains helps during the solving of school problems referring to the same issue. The study follows the basic question: What is the connection between the school mathematical word problems and the real-life experience? The connection can work in two ways – motivational and cognitive. It means that pupils are motivated to solve school problems which resemble their life experience. The experience also helps to find the best calculation procedure and check the results. The sample of our study consisted of 1,383 pupils of six grades at the Czech grammar schools. The data were collected via questionnaire on life experience and didactic test including mathematical word problems. The analysis shows that students differed in the degree of subjective proficiency in individual real-life domains and the proficiency in real-life domains positively correlates with the success in solving of word problems. However, the correlation does not exist in all age groups.

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1. Introduction

National and international surveys repeatedly show that, similarly to other European countries, mathematics is not a very popular subject among Czech students. This trend has been strongly criticized because mathematics is considered to be a crucial domain by both the professional and the general public. It is considered indispensable for the development of analytical thinking and for practical life, including success on the labour market. It is therefore logical that ways to increase the popularity of mathematics are sought for (Engeström, Miettinen, & Punamäki, 1999, Greer, 1997, Nie, & Lau, 2010). We can identify two traditional approaches in this respect. The first focuses on the teaching methods used and aims to ensure that mathematics is taught in such a way as to be interesting and fun to learn for students. The other stresses the importance of mathematics and it aims to make students understand and acknowledge the importance of mathematics for real life. Although both approaches overlap in some cases, a stronger stress is put on one or the other in most of the cases. However, both approaches alike are trying to influence the motivation of students to study school mathematics and to choose such disciplines for their future studies and work where mathematics plays an important role.

This paper follows upon the second of the described lines of tradition, which discusses the relation between mathematics and out-of-school reality in which the students live, for which they prepare or which they will enter in the future. The topic investigated by the study presented in this paper is the effect of extracurricular experiences of children on their ability to solve mathematical problems. Consequently, it also leads to the question whether children's extracurricular experiences can and should be used in the school teaching of mathematics and whether improvement of mathematical skills can be expected if such experiences are included into the school curriculum. As far as the theoretical background is concerned, our study investigates the pedagogic-constructivist paradigm and discusses its efficiency and potential limitations when applied to school mathematics.

1.1. Word problems and school mathematics

A word problem in school mathematics can be defined as: “[...] a text (typically containing quantitative information) that describes a situation assumed as familiar to the reader and poses a quantitative question, an answer to which can be derived by mathematical operations performed on the data provided in the text” (Greer, Verschaffel, & De Corte, 2003, p. 271). As follows from the definition above, the ability to find solutions to word problems is dependent on two factors. One of them are mathematical skills and the knowledge of specific mathematical operations that can be applied to the described situation. The other factor is the practical experience with dealing with identical or similar situations in reality. While the first factor may in itself be sufficient for successfully solving the problem, the second factor is more of an instrumental nature and, in most cases, it must be combined with mathematical knowledge, which it helps to structure and choose the most adequate method available. Mathematical knowledge can thus be regarded as a necessary but not sufficient condition for solving a word problem. A number of studies show that even students with good school performance in mathematics often fail in solving word problems (Schoenfeld, 2014). The reason is that they rely on the learned patterns of problem solving and ignore the limitations of these patterns when applied to specific real-life contexts.

A well-described phenomenon nowadays is the didactic contract (Brousseau, 1986). Under the influence of the didactic contract, students do not consider alternative mathematical operations as an option and they also fail to take into account the practical implications of the problem to be solved. Instead, they follow their assumptions about which mathematical operation the problem-solving exercise intends to verify and about what answers are expected by their teacher. In other words, this means that students' solutions to word problems are subject to the explicit rules of formal mathematics, but also to the implicit rules of the teaching process in the classroom. Both types of rules can be very remote from the application of mathematical operations in non-school real situations. As a consequence, the solutions offered to word problems sometimes completely contradict the laws of nature (e.g. one child walking a distance of 280 km per day, or an inflatable balloon cut in half to be distributed among a group of children) and students fail to perceive such absurd solutions as a signal of a wrong answer to the problem.

1.2. Cognitive factors

Solving word problems requires choosing the appropriate mathematical operation. In the Piagetian tradition, mathematical operations are a result of reflective abstraction on actions such as putting together and separating sets of objects. Students may understand objects as formal or real categories. Hegarty, Mayer and Monk (1995) showed that unsuccessful problem solvers solve the problem based on the numbers and keywords that they select from the word description. They call it the direct translation strategy. On the other hand, successful problem solvers develop a model of the situation from the word description and base their solution plan on this model. Such approach is called the problem model strategy. Mathematical modelling is the crucial element for understanding the solving strategies in the space between school mathematics and real life. According to Gravemeijer, the models chosen by students serve as "mediating tools to bridge the gap between situated knowledge and formal mathematics" (1994, p. 100).

If the bridge between the real-life rules presumed in word problems and formal mathematical knowledge is non-functional, students' attempts to solve the problem may fail. This failure has long been attributed to a certain type of cognitive deficit. However, this assumption has not been confirmed by recent studies. Even children who are otherwise successful in school and do relatively well in mathematics fail in solving word problems. Also in terms of formally measured intelligence, failing students are not necessarily the students with the lowest IQ scores (Wenke & Frensch, 2003). On the other hand, a link between being successful in solving school word problems and creativity, which is also connected with the IQ score, was demonstrated. Eysenck (1997) summarizes decades of creativity research by concluding that highly creative individuals have a broad range of attention and "tend to filter out fewer distracters in the environment" (p. 22). This means that creative people are able to take into account information that other people ignore as irrelevant and they can thus identify subtle patterns when solving problems which can help them discover an alternative solution to the one that, despite being apparently obvious, is wrong.

1.3. Social factors

Recent research seems to support another explanation of why a large part of students fail in solving mathematical word problems. Instead of cognitive deficit, the school and classroom culture are considered to be the main reasons (Wyndhamn & Saljo, 1997). Part of the educational culture is also the specific content and method of teaching used by each teacher, which on the one hand establishes corresponding thinking patterns and solution-solving of word problems among students but on the other hand creates anticipations of what are the teacher's expectations. This creates the already mentioned didactic contract (Brousseau, 1986) which helps students fulfil more easily the teacher's requirements, but also limits the production of new ways of thinking and solving word problems. The present research tries to map the factors relating to the process of teaching, such as teaching methods, types of word problems or teacher's expectations, and the factors on the part of students, such as the preferred solution strategies.

However, external social factors on the part of the students have been paid only little attention. At the same time, the extracurricular experience, with which the students enter the teaching process, plays – at least in the Piagetian and Vygotskian tradition – a crucial role. What could the extracurricular experience offer for solving word problems? One of the consequences is that repeated experience from a specific domain creates a good knowledge of the content and processes from that domain, and therefore understanding the situations from that specific domain when used in word problem descriptions does not require much effort. Deeper understanding can then enable departure from school sample solutions, because the student is capable of structuring and restructuring the situation. The other consequence is that repeated experience indicates that the student is interested in the given domain, and is therefore involved also emotionally. This in turn results in stronger motivation to study the relevant domain further and probably also solve word problems relating it. To provide a full picture, it should also be mentioned that apart from the described positive effects, extracurricular experience from a particular domain may also be associated with risks in the form of fixed procedures which in this case do not come from school teaching, but from practice. Another risk is the overloading of the student's operating memory in the event that the student has no matching experience from the domain to which the situation described in the word problem relates.

2. Problem Statement

As results from previous state of art, we address the connection between school formal mathematics and real-life experience with mathematical background. We consider the question how these two fields are and should be connected to support students learning the crucial. Although there are many studies on this topic, the character of connection has not yet been explained completely.

The present study researched students' results in mathematical tests including word problems with regard to their life experience in the domains to which the context of the resolved problems relates. This paper presents the first quantitative study on this topic which will be followed by a more detailed quantitative study focusing on specific mathematical word problems and by a qualitative study based on observations, interviews and focus groups conducted in three classes at different school grades. When designing the research, we intended to use the quantitative study for the initial investigation of whether

having a real-life experience with the context of a mathematical word problem is relevant for solving the problem.]

3. Research Questions

[In the first quantitative study presented in this paper, we addressed the following research questions:

- What level of experience do students of different ages have in individual real-life domains?
- Is there a relationship between having a real-life experience and the ability to solve a mathematical word problem?]

4. Purpose of the Study

[In our study, we focused on the extracurricular experience of students and its relationship to the success rate in solving word problems. Specifically, we were interested in whether students solve word problems better if these relate to the life domains (e.g. transport) in which they have a greater amount of experience that makes them confident they have good understanding of the domain. We focused on the following 12 domains: 1. Public transport(trains and buses, timetables); 2. Means of transport(cars, motorbikes, speed...); 3. Sports (doing or watching sports); 4. Organizing group activities (e.g. creating and leading teams); 5. Household chores (cooking, cleaning...); 6. Shopping (prices, discounts, food, clothing...); 7. Building and construction (interest in constructions, building sets...); 8. Creativity (drawing, embroidery, jewellery making); 9. Nature, animals, plants (trips, interest in animals and plants and caring for them); 10. Music (listening, own playing); 11. Tourism (orientation in maps, distances...) 12. Communicating in an unfamiliar environment (e.g. asking for directions)]

5. Research Methods

5.1. Study design

To investigate the students' abilities to solve mathematical word problems, a survey was carried out using a written didactic test and a written questionnaire. The questionnaire was used to collect answers to research question 1. The research question 2 was answered by using a combination of data collected from questionnaires and test results.

5.2. Participants

The participants of the study were students of six grades at four Czech grammar schools. The schools and the students take part in a three-year study of which the present study is a part of. We obtained informed approvals from students and/or their parents. The sample included in the present study consisted of 1,383 students who were asked to complete the questionnaire and take part in the didactic test. The sample covered six grades with the following distribution: 301 students (154 females and 147 males) from the 4th grade at the average age of 9.47 years (SD=.518); 300 students (150 females and 150

males) from the 5th grade at the average age of 10.55 years (SD=.538); 243 students (125 females and 118 males) from the 6th grade at the average age of 11.44 years (SD=.506); 180 students (88 females and 92 males) from the 7th grade at the average age of 12.54 years (SD=.553); 209 students (103 females and 106 males) from the 8th grade at the average age of 13.47 years (SD=.538); 150 students (88 females and 62 males) from the 9th grade at the average age of 14.50 years (SD=.528). The ratio between the male and female students was approximately 1:1 (variation from 48% to 51%), which corresponds to the national statistics, except for the 9th grade where the proportion of girls was 58%, which is above the statistical average.

5.3. Measures

5.3.1. Didactic test

Didactic test in mathematics was used for initial mapping of students' knowledge. The test for each grade followed a similar structure – it comprised of four different types of word problems and additional three to five numerical exercises to verify the knowledge of the mathematical operations necessary for solving the word problems in the first part of the test. The aim of the test was to determine how students coped with solving typical word problems and, in cases of failure, to determine whether the cause of failure was a lack of understanding of the underlying mathematical operation or whether it was the context of the word problem.

The tests were designed to cover the subject matter that had already been covered during the teaching classes and that corresponded to the textbooks that had been used. The tests were taken by the students under supervision by trained examiners. The time limit for completing the test was set at 35 minutes ensuring that the test could be taken in the course of one teaching lesson. The majority of students succeeded to complete the test within the specified time limit.

All test items were open and it was therefore necessary to first evaluate the students' solutions and classify them using a common classification key. Subsequently, the results were recorded in an electronic form. As the tests for all grades were designed to contain certain common items (called the anchor items), it was possible to use analytical methods based on both the classical test theory and the item-response theory.

5.3.2. Questionnaire

The student questionnaire comprised 20 items. The first part of the questionnaire collected identification data, such as gender, age and family background. The second part focused on the objective and subjective success in mathematics and on the popularity of mathematics in comparison to other subjects. The third part consisted of items relating to the teaching practices, such as the frequency of solving word problems in mathematics classes, marking, homework, ancillary solution strategies etc. The fourth part focused on the reading experience. The fifth part was an abridged version of the mathematics self-efficacy scale.

Finally, the sixth part examined the level of proficiency in 13 real-life domains (see the list above) that had been selected on the basis of previous qualitative interviews. The mathematical problems assigned during individual phases of the research will relate to these domains. For each of the domains,

the students provided an estimate of their proficiency in the relevant domain (rating their knowledge and experience) on a scale from 1 (highest) to 5 (lowest).

The questionnaire was validated by a pilot study. With regard to its scope, the questionnaire required approximately 25–30 minutes for completion, and so it could be completed in the course of one teaching lesson similarly as the test. The questionnaires were completed 2–3 weeks after taking the didactic test. All questionnaire items were closed or semi-closed, so the students' responses could be easily captured in electronic form.

During the first study, which is presented in this paper (the first didactic test), the following contexts were used in word problem descriptions: 1. Means of transport (specifically, speed of a car and a bus); 2. Shopping (specifically, sum of prices of goods and calculation of a discount); 3. Constructions and technical principles (specifically, calculation of weight of different materials and composition of materials); 4. Relationships between people and their organisation (specifically, establishment of cooperatives and determining age in a relationship); 5. Tourism (specifically, finding routes on a map). Responses from these five domains were then used to calculate the index value (an aggregate value) that could reach between 5 to 25 points.

6. Findings

6.1. Proficiency in real-life domains

The word problems included in the tests referred to five different domains, for which the level of proficiency was established in the questionnaire. To assess the proficiency in these five domains, an aggregate index was calculated using the following rating scale: 5–10 (good proficiency), 11–15 (uncertain proficiency), 16–25 (poor proficiency). In the total sample, 35% of students reported good proficiency and 20% poor proficiency in the assessed domains. Most frequently, the students mentioned the “shopping” domain as a domain with good proficiency (67%), followed by the domains of “means of transport” and “relationships between people and their organization” (each 60%), smaller “tourism” (49%) and “constructions and technical principles” (40%).

At higher grades, the subjective assessment of level of proficiency in individual domains decreased. Although students certainly become more experienced in all of the assessed domains with increasing age, their self-assessment does not correspond to that. This is an outcome that was to be expected, because it corresponds to the nature of cognitive development and to the increased self-criticism with increasing age. The only exception is the domain of shopping, in which the differences between the grades are statistically insignificant and do not exhibit any trend. On the other hand, the biggest drop was identified in the domain of “constructions and technical principles” where the average rating on a scale from 1 to 5 increased from 2.3 to 3.6 (in other domains, the changes in rating did not exceed 0.5 points).

In all domains with the exception of “relationships between people and their organization”, there was a statistically significant difference between the answers provided by girls and boys ($p < .001$). In the case of means of transport, tourism and constructions and technical principles, boys reported higher level of proficiency in the domain, while in the case of the shopping domain, it was girls. Also, the aggregate value of the index for all five assessed domains showed a statistically significant difference between the

responses of girls and boys where boys reported better levels of proficiency – $M(\text{boys})=11.82$ ($SD=3.83$), $M(\text{girls})=12.73$ ($SD=3.65$), $t(1383)=4.493$, $p<.001$. In individual grades, gender-related differences in the value of the index were observed only in grades 4, 7 and 9 ($p<.05$). However, differences in the three key domains most frequently used in world problems (i.e. means of transport, shopping, and constructions and technical principles) were observed in all grades ($p<.01$) with the exception of “shopping” in grade 7.

6.2. Proficiency in real-life domains and success in solving word problems

The didactic test contained 4 word problems including the related mathematical operations. The average success rate in the test measured by the aggregate score was 66.7% ($SD=0.23$). The success rate of students at higher grades was relatively lower – the students at grade 4 achieved an average 70% success rate while in the case of students at grade 9 it was just 56%. However, results across individual grades fluctuated, so no clear trend suggesting declining success rate with increasing age could be conclusively evidenced. Regardless of age, we further measured the differences in success rate between girls and boys. The average success rates of boys and girls were 69.2% and 64.2%, respectively. These differences are statistically significant, $t(1383)=-4.003$, $p<.001$. However, the gender-based differences were statistically significant only in grades 4, 7, and 9, in each case in favour of boys.

We correlated the index measuring proficiency in the five assessed domains with the success rate in the test. Pearson correlation coefficient reached the value of $-.162$, which is a significant correlation with $p<.001$. Further, we divided the index measuring domain proficiency into three categories (good, uncertain and poor proficiency) and we calculated the average success rates in solving the didactic test for each of these categories. Students with good proficiency in the assessed domains achieved an average success rate of 71.2% in the test ($SD=.203$), students with uncertain proficiency reached 65.5% ($SD=.234$), and students with poor proficiency 61.2% ($SD=.259$). The difference in success rates between students with good and poor proficiency in the domains was statistically significant, $t(763)=5.896$, $p<.001$. Students with little real-life experience scored on the average worse in the test in comparison to students with rich real-life experience.

A significant difference between the group with good and poor domain proficiency was also seen in the girls’ and boys’ subgroups. However, when conducting a more detailed analysis of the results for individual grades, we identified an overall difference only in grade 8, $t(114)=2.26$, $p<.05$. In grade 9, significant difference was identified only in the boys group, $t(59)=2.45$, $p<.05$. There was no significant difference observed between boys and girls with different levels of domain proficiency in other grades, either.

6.3. Discussion

Our study focused on the relationship between subjective proficiency in specific real-life domains and solving of mathematical word problems that used the relevant domains as their context. On the basis of previous research, we assumed that students with good proficiency in a specific domain would be able to solve problems from the relevant domain with better success, because they are more motivated and also have models available allowing them to more successfully restructure their formal mathematical knowledge (Gravemeijer, 1994). This assumption corresponds to the Piagetian tradition of educational

constructivism, according to which establishing a dialogue between the student's pre-concepts and concepts built at school are key to effective learning.

However, the results of our study support the above hypothesis only partially. We found that students differed in the degree of subjective proficiency in individual real-life domains. It is then possible to compare their success rate in solving a didactic test against this subjectively perceived proficiency. Also, the average success rate of the entire set as well as in individual age and gender subgroups varies sufficiently enough to substantiate making a comparison with regard to real-life experience. Nevertheless, the very nature of the relationship between solving school problems and proficiency in real-life situations is unclear. A weak, although statistically significant correlation between the two variables has been confirmed. Also, an independent analysis of individual subgroups with different real-life experiences showed that there were differences in the success rates in tests. This would therefore suggest that proficiency in real-life domains may have positive impact on the ability to solve school problems. However, this conclusion does not apply to all of the grades. We thus could not fully accept the interpretation presented above. However, because non-existence of a correlation between the two variables has not been confirmed, we consider it necessary to invest further effort into conducting further studies in this field. Nevertheless, it is evident from the results of our study as well as from previous research on which we follow-up that in solving mathematical word problems a number of factors relating to the school and class culture interfere (Wyndhamn, & Saljo, 1997) including extra-curricular factors.

Our study faced several limitations that we consider necessary to eliminate in our future research. The main limitation has been the fact that we assessed only the overall success rate in tests, not the success rates in individual test items. Performing an analysis at the level of individual items would allow us making a more precise comparison between the context of the word problem and the proficiency in the related real-life domain. It is not necessary to use only a questionnaire to evaluate experience with real-life domains. Using a questionnaire requires the use of relatively high-level, general formulations that are somewhat difficult to grasp for the younger students. Instead, a question following immediately after each specific word problem, could be used. It would thus be possible to attach a question after each word problem – Have you experienced a similar situation in your real life?

The second limitation is the lack of information about the ways in which word problems are taught and in which their solving is practiced during individual teaching lessons. It is likely that the solutions offered by students are influenced by the didactical contract (Brousseau, 1986), which can induce poorer students to trying to offer solutions corresponding to their idea of the expected performance rather than productively searching for optimal solutions to the problem. Although we included only typical word problems in the test which verified the subject matter already covered during the lessons and with which the students had earlier worked in their textbooks, we lack detailed information from teachers. In future studies, we therefore recommend asking the teachers to assess the externally selected word problems with regard to the students' achieved level of knowledge and, optimally, we would also like to ask them to describe the solution method they expect to be offered by the majority of students.

A further step towards deepening the analysis should be the inclusion of information on the objective and subjective success of students in mathematics, i.e. about their school marks and their self-assessment. On the basis of our complementary analyses we assume that the use of extracurricular experience takes place differently for students with different mathematical abilities. It is likely that the

fact whether extracurricular experience acts as a support or, on the contrary, as a barrier for resolving formal mathematical problems is dependent on how strong (their real skill and/or convictions) the students are in mathematics.

7. Conclusion

Without trying to belittle the importance of other school subjects, of which each contributes to the comprehensive development of students, we consider mathematics to be truly one of the key school subjects. It helps developing a specific type of thinking and, as such, it plays a unique role in school. Mathematical knowledge and skills are a prerequisite for success in a number of professional areas and they have also use in everyday life. For these reasons, it is necessary to look for ways to support students in the most efficient learning.

If students consider word problems to be a particularly difficult subject matter, then attention should be paid (among others) word problems specifically. One of the most frequent reactions to students' failures and lack of interest in mathematics and in word problems specifically is their weak interconnection with real life. This claim (highly attractive among lay public) needs to be verified by research and our study provides a small contribution in this respect. However, based on the results of our study it is not possible to confirm that there is a clear positive link between real-life experiences and success rate in solving word problems relating to a similar context. Although our study suggested the such relationship exists, it has not been proven neither as strong nor as present in all six studied school grades. This means that more research is needed to address the above-described weaknesses of our study.

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