The European Proceedings of
Social & Behavioural Sciences
EpSBS

Future Academy ISSN: 2357-1330

http://dx.doi.org/10.15405/epsbs.2017.05.02.167

Edu World 2016
7th International Conference

METHODS OF DEMONSTRATING CONCURRENCE LINES IN A TETRAHEDRON

Costică Lupu (a)*

* Corresponding author

(a) Faculty of Sciences, Department of Mathematics-Informatics and Education Sciences, "Vasile Alecsandri" University of Bacău, Str. Mărăşeşti 157, 600115, România. E-mail address: costica_lupu@yahoo.com

Abstract

This article discusses the advantages of teaching-learning Geometry with the help of the new educational technologies to motivate students and undergraduates, which may lead to building focus, interactivity and feedback during lessons. The 3rd-year students from the Department of Mathematics, "Vasile Alecsandri" University of Bacău, also attending the courses of the Pre- and In-Service Teacher Training Department, have experimented the relevance of the new technologies in teaching and learning Geometry, by applying a new approach to the design and conduct of a lesson of Geometry.

The research was conducted during the 2014-2015 school year, involving two groups, each of them comprising 132 students: 66 children, from 3 classes having 22 students on the average each, experimental group – the 8th A grade from "Octavian Voicu" Middle School, Bacău, and 66 children, from 3 classes having 22 students on the average each, a control group – the 8th B grade from "Miron Costin" Middle School, Bacău and demonstrated the efficiency of teaching Geometry the concurrence of relevant lines in a tetrahedron by using computers in solving problems and promoting Geometry knowledge, building active thinking and competences, as well as skills in representing plane and space geometrical figures.

Keywords: AEL laboratories; graphical representation; concurrence; tetrahedron; experimental research.

http://dx.doi.org/10.15405/epsbs.2017.05.02.167
Corresponding Author: Costică Lupu
Selection and peer-review under responsibility of the Organizing Committee of the conference
eISSN: 2357-1330

1. Introduction

In order to make the economy of the European Union a most competitive one at the global level, it is necessary to build a strong foundation for the education of future generations. In order to facilitate the success of an economy, it is required that the physical persons owning thorough knowledge, key competences and who are consistent with the latest technologies should firmly apply their expertise and information to today's society, as shown in the Lisbon Strategy.

http://dx.doi.org/10.15405/epsbs.2017.05.02.167
Corresponding Author: Costică Lupu
Selection and peer-review under responsibility of the Organizing Committee of the conference
eISSN: 2357-1330

The Lisbon Strategy is a key European orientation that plans to implement a more relevant use of information and communication strategies in the classroom. The gaps at the educational level are very serious. The Romanian university education should undertake a European approach in order to create new and innovating concepts, pedagogical and didactic strategies, to improve teaching and learning (Cerghit, 2002).

The students should build key skills, including digital literacy, foreign languages and – in our context – Mathematics, through professional training. University training should take into consideration the students' team-work needs within a strong European partnership and focus on improving teaching-learning strategies.

The students' initial training aims at a fundamental improvement of teachers at a European level. By introducing the innovative ideas of ICT, present and future teachers should ensure a modern professional education. This is a prerequisite for building and supporting a highly motivated society, with a high level of awareness in Europe. (Lupu, 2013).

Building digital educational content is essential for the use of modern information and communication technologies in mathematical education (both in schools and universities). We shall present and evaluate the experience in using modern educational software in and outside the classroom.

The teachers from Romanian schools are interested in using AEL laboratories and ICT during lessons, as well as dynamic mathematical software for teaching and learning. Also, there is required the use of the interactive board in everyday lessons, hence every school should have an interactive board. The relevance of the role of teachers as partners of students was highlighted by the use of laboratory-digitalised environments in mathematical training.

A concern of university teachers is to organize courses or workshops on the use of computers in teaching Mathematics, with the aim of training middle and high-school teachers in this respect. Mathematical software represents a highly supportive component for encouraging individual learning and solving Geometrical problems, by providing students and undergraduates with tools that may be individually used inside, as well as outside the classroom, in the initial training of teachers.

During the Teaching Practice, the students and pupils from the research group used AEL programs to graphically represent geometrical figures, mainly tetrahedrons, highlighting relevant lines and concurrence properties.

The experimental research consisted in evaluating some initial, formative and final tests that highlighted the importance of using computers, as well as acquiring the relevant lines in a tetrahedron, with their concurrence properties.

2. Theoretical Aspects. The Concurrence of Relevant Lines in a Tetrahedron

We shall analyse some concurrence problems of the following relevant lines in a tetrahedron: the medians (the segment connecting a point with the centre of gravity of the opposite side), the bimedians (the segment uniting the middle points of two opposite edges), the perpendiculars to the sides of the tetrahedron on their centres. There rose the following problem-situations: Are the heights in a tetrahedron concurrent? Is there a condition if and only if, that ensures that the heights in a tetrahedron are concurrent? (Lupu, & Săvulescu, 1999, p.84)

http://dx.doi.org/10.15405/epsbs.2017.05.02.167
Corresponding Author: Costică Lupu
Selection and peer-review under responsibility of the Organizing Committee of the conference
eISSN: 2357-1330

Problem 1. Let M be the middle of the edge CD of a tetrahedron ABCD, and I∈BM and J∈AM the points of the centres of gravity of the sides BCD and ACD. Prove that the medians AI and BJ are concurrent. (Lupu, & Săvulescu, 2000, p. 127)

Demonstration: Because I and J represent the centre's of gravity of sides BCD and ACD, they will be located on medians BM and AM at $\frac{1}{3}$ from CD and $\frac{3}{3}$ from vertex B and A. In triangle ABM, $IJ \parallel AB$ and let there be $AI \cap BJ = \{G\}$, then $\frac{\mathrm{IG}}{GA} = \frac{JG}{GB} = \frac{IJ}{AB} = \frac{MI}{MB} = \frac{MJ}{MA} = \frac{1}{3}$. The medians of the tetrahedron, being coplanar two by two, are also concurrent two by two, therefore they have a common G point that divides each median according to the relation1/3 and is called the centre of gravity of the tetrahedron. Regarding the concurrence of medians, there is demonstrated that: In a tetrahedron ABCD, the medians MP, NQ are RS are concurrent. (M∈BC, MB ≡ MC ; N∈CD , NC ≡ ND ; P∈AD , PA ≡ PD ; Q∈AB , QA ≡ QB ; R∈AC, RA ≡ RC ; S∈BD , SA ≡ SO).

For demonstration, observe that the quadrilateral MNPQ is a parallelogram, and QN and MP are concurrent, being diagonals in this parallelogram and cut into half. By analogy, RS cuts the middle of QN and MP. We may further demonstrate that the concurrence point of the medians is also the centre of gravity G, because the median MP, for example, is the median of the median plane (MAD) and of the median plane (NBD), and since the gravity centre belongs to each of the median planes of the tetrahedron, it results that G also belongs to each median. (Peligrad, Țurcanu, & Popa , 2015).

Regarding the concurrence of the perpendiculars on the centres of the circles circumscribed to the lateral sides, we will first solve the fact that the bisecting planes of the sides of a triangle have a common line. Building the bisecting planes α and β of sides AB and BC from ΔABC we will have $\alpha \cap \beta$ = PO and $PA \equiv PB \equiv PC$. Line PO of the intersection of the bisecting planes of triangle ABC is the median in space of this triangle and every point on it is equally distanced from the triangle tips.

Then, we demonstrate that the six bisecting planes of the sides of a tetrahedron are concurrent in a point that is equally distanced from the vertices of the tetrahedron. If m1 is the median in space of the plane of triangle ABC and μ the bisecting plane of the edge AD where O = $m_1 \cap \mu$, we have: $OA \equiv OB \equiv OC \equiv OD = R$ which shows that O also belongs to the other bisecting planes.

The sphere with the centre O and radius R is called the sphere circumscribed to the tetrahedron. Are the heights concurrent? The answer is no. We may illustrate it with an example: in the tetrahedron DABC with the basis an equilateral Δ ABC and DA ⊥ (ABC), the heights BM, CN and DA are not concurrent. Regarding the fact that the heights of a tetrahedron should be concurrent, we shall firstly define the orthocentric or orthogonal tetrahedron (a term coined by Steiner in 1927), namely, a tetrahedron is orthocentric if its opposing edges are perpendicular.

Lemma: If in the tetrahedron ABCD two pairs of opposing edges are perpendicular, then the other two edges are also perpendicular.

Demonstration: Let there be AB ⊥ CD and BC ⊥ AD. Draw AE ⊥ DC and AF ⊥ BC. It results CD ⊥ (ABE) and BC ⊥ (ADF). If AA′ = (ABE)∩ (ADF) then AA′⊥ (BCD). But CA′⊥BD and therefore

BD⊥(ACA′), it results BD ⊥ AC. Then, demonstrate the theorems.

Theorem 1: The heights of a tetrahedron are concurrent if and only if the tetrahedron is orthocentric.

http://dx.doi.org/10.15405/epsbs.2017.05.02.167
Corresponding Author: Costică Lupu
Selection and peer-review under responsibility of the Organizing Committee of the conference
eISSN: 2357-1330

Demonstration: We demonstrate that the heights from A and D are concurrent when AD⊥BC. Let A′ and D′ be the orthogonal projections of points A and D on planes (DBC) and (ABC) and AA′ and DD′ concurrent, AA′⊥(DBC)⇒AA′⊥BC and DD′⊥(ABC)⇒DD′⊥BC. Therefore, BC being perpendicular to plane (AA′,DD′), it is therefore also perpendicular to AD. Reciprocally, if AD⊥BC, let F be the foot of the perpendicular from A to BC. From BC⊥AD and BC⊥AF it results that BC⊥(AFD)⇒BC⊥AA′ . Analogously, BC⊥DD′. Since AA′⊂(AFD) and DD′⊂(AFD) it results that AA′ and DD′ are heights in triangle (AFD), therefore are concurrent. Hence, it results that if the heights AA′ and DD′ are concurrent, then the other two heights BB′ and CC′ are also concurrent. Indeed, AA′∩DD′=∅ ⇒ AD ⊥ BC therefore BB′∩ CC′=∅. (Lupu, 2014, p. 135).

We may demonstrate that if heights AA′ and BB′ are concurrent in H, then the common perpendicular of edges AD and BC contains the point H, because if the plane (AA′,DD′) cuts BC in F, it results that FH is the third height, therefore FH⊥AD; since BC⊥(AFD) ⇒ FH⊥BC. By analogy, we may demonstrate that if the heights AA′ and DD′ are cut in point H and BB′ and CC′ in H′≠ H then HH′ is the common perpendicular of the opposing edges AD and BC.

From the three conditions of perpendicularity of the orthocentric tetrahedron, it results the concurrence of heights. Another particularly interesting theorem for analysis is:

Theorem 2: A tetrahedron is orthocentric if and only if: $AB^2 + CD^2 = AC^2 + BD^2 = AD^2 + BC^2$.

Demonstration: We demonstrate, first of all, that the heights from A and D of tetrahedron ABCD are secant if and only if$AB^2 + CD^2 = AC^2 + BD^2$. Let there be F and F_1 the orthogonal projections of points A and D to BC. There occur the relations: $AB^2 - AC^2 = (AF^2 + BF^2) - (AF^2 + FC^2) =$

$BF^2 - FC^2 = (BF + FC) \cdot (BF - FC) = BC(BF - FC)$ (1); $BD^2 - CD^2 = ({DF_1}^2 + {BF_1}^2) -$ $(DF^2 + F_1C^2) = {BF_1}^2 - {CF_1}^2 = (BF_1 + CF_1) \cdot (BF_1 - CF_1) = BC(BF_1 - CF_1)$ (2).

It can be observed that AA′ and DD′ are secant if and only if F coincides with F_1. From the previous equalities with $F = F_1 \Rightarrow$ $AB^2 - AC^2 = BD^2 - CD^2 \Rightarrow$ $AB^2 + CD^2 = BD^2 + AC^2$. Reciprocally $AB^2 + CD^2 = BD^2 + AC^2 \Rightarrow BC(BF - FC) = BC(BF_1 - F_1C)$ therefore $BF - FC =$ $(BF + FF_1) -$

$-(FF_1 + FC) = 2FF_1 = 0$ therefore F and F_1 coincide. Demonstrating analogously and taking into consideration the concurrence of all the other heights, it results: $AB^2 + CD^2 = AC^2 + BD^2 = AD^2 + BC^2$. We can also easily deduce the fact that the tetrahedron ABCD is orthocentric if and only if its three bimedians are congruent (Lupu, 2015, p. 850).

3. Research Description

3.1. Research Objectives

By using the proper strategies in teaching Mathematics, we may achieve a higher learning performance at 8th-graders. The undergraduates were preoccupied with equipping students with thorough skills and abilities in using mathematical software, thus optimizing the process of learning Mathematics. In this respect, there were established several objectives for guiding and orienting the whole investigation: - theoretical classification of the contexts approached from the perspective of the theme; - initial evaluation of the level of mathematical knowledge, skills and abilities of students, at the beginning

http://dx.doi.org/10.15405/epsbs.2017.05.02.167
Corresponding Author: Costică Lupu
Selection and peer-review under responsibility of the Organizing Committee of the conference
eISSN: 2357-1330

of the 1st semester, in the 2014-2015 school year, at two 8th-grade classes; - evaluation of computer skills; - final evaluation of the level of knowledge following the application of the progress factor, respectively the concurrence of relevant lines in a tetrahedron; - formulating the conclusions of the study (Landsheere 1979).

3.2. Research Hypothesis

To achieve these objectives and verify the role of solving and composing problems in building the creative potential of small students, we have formulated the following hypothesis: if we use the new technologies in teaching-learning Geometry in the concurrence of relevant lines in a tetrahedron, then we shall contribute to building intellectual skills and the creative potential of the middle-school student, as well as enhance the concurrence of relevant lines in a tetrahedron.

3.3. Research Methods

In our research, we have used the following methods: observation; conversation; evaluation test; analysis of activity products. We have used the formative type of psycho-pedagogical experiment in order to verify the impact of using active-participative methods and modern means and techniques for teaching-learning-evaluating elements of Geometry related to the methods of demonstrating concurrence lines in a tetrahedron. The statistical method was used to process, analyze, systematize and interpret the data obtained following the measurements. The data from the synthetic tables were graphically represented through: histograms, frequency polygons and round diagrams (Cristea, 2003).

3.4. Research Stages and Sample

The research was conducted during the 2014-2015 school year, involving two groups, each of them comprising 132 students: 66 children, from 3 classes having 22 students on the average each, experimental group – the 8th A grade from "Octavian Voicu" Middle School, Bacău, and 66 children, from 3 classes having 22 students on the average each, a control group – the 8th B grade from "Miron Costin" Middle School, Bacău.

The basic method used was the pedagogical, observational-formative experiment, applied through the method of the tests. The research involved several stages:

1. The observational stage that covered the interval October 15th – October 30th. At this stage, there were applied tests for identifying the initial level of the students' digital and intellectual skills.

2. The ameliorative stage, conducted over the 1st and 2nd semester, November 1st – May 28th. At this stage, there were organized differentiated training activities, aimed at improving digital competences by using mathematical software (progress factors within the conducted experiment). The analysis of the results from the evaluation tests enabled the implementation of differentiated, ameliorative pedagogical measures. Besides the activities conducted for this purpose, at the lessons from the optional discipline Educational software, during the sequences of independent activities, the students received individual or group tasks. Following these activities, the students made visible progress.

http://dx.doi.org/10.15405/epsbs.2017.05.02.167
Corresponding Author: Costică Lupu
Selection and peer-review under responsibility of the Organizing Committee of the conference
eISSN: 2357-1330

3. The stage of final evaluation covered the interval June 1st – June 10th and was aimed at retesting the digital skills of students, as well as their potential for using the computer in the graphical representation at the end of the pedagogical experiment.

In our ameliorative experimental research, obtaining evaluation information based on tests with pedagogical objectives requires coverage of the following stages: - applying the test and obtaining students' answers, which are evaluated; - comparing each answer with the standard solution and giving the corresponding score for correct solutions; - giving school marks by applying the criteria of converting scores into marks; - interpreting the results and adopting ameliorative measures (Dewey, 1992).

4. Research Results

This study is a continuation of our previous articles (Lupu, 2014 & Lupu, 2015) where we have demonstrated that informational technologies hold a privileged position within the set of the factors responsible for the students' school success and how using computers and active, cooperation methods during Geometry lessons has a positive effect upon students, supporting the building of digital, communication and team-work skills.

The analysis of the analytical and synthetic tables, the histogram, the frequency polygon and the circular diagram generated the following conclusions regarding the initial evaluation: the experimental group has achieved the following results: of the 66 evaluated children, 29 obtained the mark of very well, representing 44% of them, 24 children obtained the mark of well, meaning 36%, and 13 children obtained the mark sufficient, representing 20% of the participants; the control group achieved the following results: of the 66 evaluated children, 29 obtained the mark very well, representing 44% of them, 26 children obtained the mark well, namely 40%, and 8 children obtained the mark sufficient, representing 12% of the participants, whereas 3 children – 4% got insufficient.

The application of the initial test enabled the identification of the students' gaps and the extent of these gaps, the prolonged emphasis on the concurrence of relevant lines until all the students have reached a corresponding training level. In Table 1 and frequency polygon 1, shows the comparative analysis of the initial evaluation for the two groups.

At the current, formative stage, in the experimental class the focus was particularly on issues related to the use of computers in the graphical representation of planes and geometrical figures, on solving and demonstrating problems.

Thus, there were regularly applied, during lessons of Mathematics, formative evaluation tests. Thus, the students solved tests based on the hypothesis that the permanentization of immediate control, as an extrinsic motivational situation, would build the motivation needed in building digital skills, imagination and creativity by solving problems through the use of the computer in graphical representation.

The formative evaluation tests applied in lessons of Geometry enabled the immediate identification of the students' learning difficulties. In order to eliminate mistakes, the activity was differentiated. Following the analysis of tests, there were presented the operational objectives that were not achieved by students, so that they may be aimed at in the proposed recovery activities.

http://dx.doi.org/10.15405/epsbs.2017.05.02.167
Corresponding Author: Costică Lupu
Selection and peer-review under responsibility of the Organizing Committee of the conference
eISSN: 2357-1330

Table 1. Comparative analysis of the initial evaluation for the two groups

Marks	Experimental group	Control group
Very well	29	29
Well	24	26
Sufficient	13	8
Insufficient	0	3

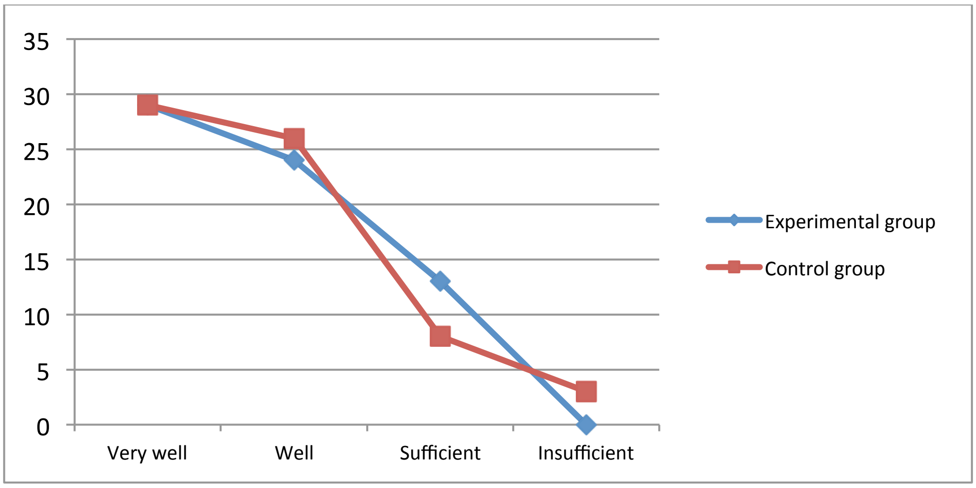

Fig. 1. Frequency polygon – Comparative analysis of initial evaluation

The analysis of the synthetic table no. 2, the histogram and the frequency polygon reveals the fact that, in the final evaluation: - for the experimental group, the results were the following: of the 66 evaluated children, 37 obtained the mark very well, representing 56% of them, an increase by 12% , 26 children obtained the mark well, their percentage also being 40 %, and 3 child got the mark S (sufficient), representing 4% of the participants; - for the control group, the results were the following: of the 66 evaluated children, 29 obtained the mark very well, representing 44% of them, 26 children obtained the mark well, their percentage also being 40%, 8 children got the mark sufficient, representing 12 % of the participants, and 3 child insufficient, representing 4 % .

Calculating the average between the two tests (initial and final), and drawing a comparison between the two groups, there may be observed an increase in the school performance at the experimental group, compared with the control group.

The comparative analysis of the histogram and frequency polygon no. 2 reveals the progress recorded by the experimental group at the end of the experiment. Calculating the average between the initial and final evaluation, there were obtained the following results.

http://dx.doi.org/10.15405/epsbs.2017.05.02.167
Corresponding Author: Costică Lupu
Selection and peer-review under responsibility of the Organizing Committee of the conference
eISSN: 2357-1330

Table 2. Shows the comparative analysis of the initial evaluation for the two groups

Marks	Experimental group	Control group
Very well	37	29
Well	26	26
Sufficient	3	8
Insufficient	0	3

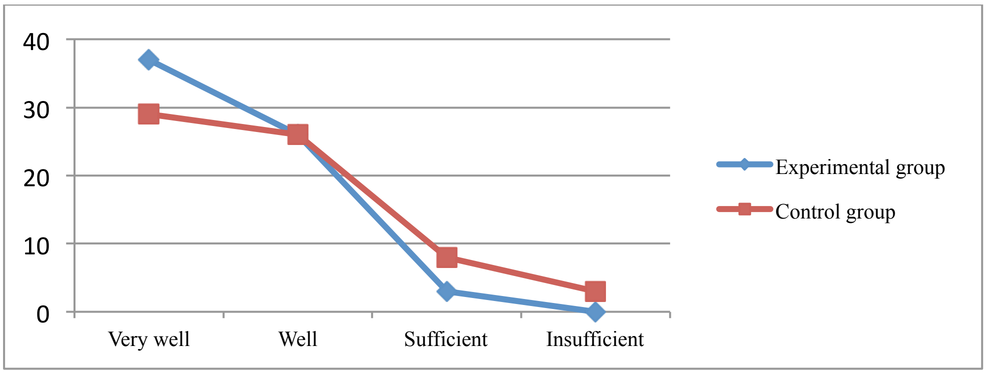

Fig. 2. Frequency polygon – graphical representation of the comparative analysis of results

5. Conclusions

At the theoretical level, there were created the premises for conceptualizing the teaching strategies for solving problems of Geometry in the concurrence of relevant lines in a tetrahedron. The analysis of the experimental test from the two classes supports the formulation the following conclusions: - the informational technologies hold a privileged position within the set of the factors responsible for the students' school success; - using computers and active, cooperation methods during Geometry lessons has a positive effect upon students, supporting the building of digital, communication and team-work skills; - the efficient use of AEL laboratories and the use of software characteristic of geometrical representation, as well as active-participative methods constitutes a challenge, both for students and the teacher. Following the application of the ameliorative experiment by using the method of the tests, as well as of active, cooperation methods in teaching, it was found that their use with a specific purpose, at the right moment, may generate satisfying results, such as: - the students overcame their communication blockages; - they built skills in solving Geometry problems; - they manifested proper behaviour towards their colleagues and group; - there was better cooperation among children, these became more tolerant; - there was a combination of types of work (frontal, group, individual), which created great possibilities for the multiple and diverse activation of students; - the students learnt that in order to achieve a group task, they need one another.

The results obtained by the students validated the research hypothesis. The use of modern technologies in the activity of solving Geometry problems contributes to optimizing learning and rendering it efficient, stimulating the students' intellectual and creative potential, obtaining performances according to age and individual particularities.

http://dx.doi.org/10.15405/epsbs.2017.05.02.167
Corresponding Author: Costică Lupu
Selection and peer-review under responsibility of the Organizing Committee of the conference
eISSN: 2357-1330

In conclusion, we may say that in order to achieve quality education and obtain the best results, we should apply the project "InnoMathEd – Innovations in Mathematics" and combine traditional and modern methods in the teaching-learning-evaluation of Mathematics.

References

Cerghit, I. (2002). Alternative and complementary training systems. Structures, styles and strategies, *Aramis Publishing*, Bucharest.

Cristea, S. (2003). Science education. General theory of education. *Letter Publishing Group / Point International*, Chisinau, Bucharest.

Dewey, J. (1992). Foundations for a science of education, Didactic and Pedagogical RA, Bucharest.

Landsheere, V. & G. (1979). Defining learning objectives, (trans.), Didactic and Pedagogic Publishing House, Bucharest.

Lupu, C. & Savulescu, D. (1999). Teaching Mathematics. Manual and college teaching class XI, Publishing House, Parallel 45, Pitesti.

Lupu, C. & Savulescu, D. (2000). Teaching Geometry, Publishing House Parallel 45, Pitesti.

Lupu, C. (2013) Demonstration by Reducing to Absurd , Procedia Social and Behavioral Sciences, Vol. 76, pp. 465-469. http://www.sciencedirect.com/science/article/pii/S1877042813006903.

Lupu, C. (2014) The Model Object-product-cognitive Operation Through Mathematical Education, Procedia - Social and Behavioral Sciences, Vol.163, pp. 132 – 141, http://www.sciencedirect.com/science/article/pii/S187704281406399X

Lupu, C. (2015), Methods of Demonstrating the Collinearity of Points in Space, Procedia - Social and Behavioral Sciences, Vol. 180, pp. 847-853, http://dx.doi.org/10.1016/j.sbspro.2015.02.223.

Peligrad, S., Ţurcanu, A., & Popa, C. S., (2015), Final evaluation standard tests. Class VII. Mathematics, Publishing House Parallel 45, Pitesti.