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# Minimal and maximal representation of degressively proportional allocation

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**Abstract**

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Boundary conditions are a significant element of a process developing the degressive proportionality principles. With regard to voting issues, this is equivalent to determining a minimum and a maximum representation, and a size of the whole assembly. In case of a practical problem, the choice of these numbers is evidently constrained. The political conditions as well as the necessity of ensuring efficient functioning of the elected body significantly restrict a set of all possible alternatives. The paper analyzes the feasibility of boundary conditions under a given minimum and maximum representation.



**Keywords:** Elections, fair division, European Parliament

## 1. Introduction

The Treaty of Lisbon (Laslier, 2012) introduced a principle of degressive proportionality of goods and burdens as a legal norm for the first time. The principle was adopted as a rule of distributing mandates to the European Parliament among the member states. The respective provision (The Treaty of Lisbon, article 9A) reads as follows (Treaty, 2010): "*The European Parliament shall be composed of representatives of the Union's citizens. They shall not exceed seven hundred and fifty in number, plus the President. Representation of citizens shall be degressively proportional, with a minimum threshold of six members per Member State. No Member State shall be allocated more than ninety-six seats*".

Additional explanations that interpret the notion of degressive proportionality can be found in the resolution titled "Proposal to amend the Treaty provisions concerning the composition of the European

Parliament" (Lamassoure, & Severin, 2007). The two statements contained there, i.e. "the larger the population of a country, the greater its entitlement to a large number of seats" and "the larger the population of a country, the more inhabitants are represented by each of its Members of the European Parliament", allow to rigorously define this principle in the language of mathematics: a positive sequence $s_1, s_2, ..., s_n$ is degressively proportional with respect to $0 < p_1 \leq p_2 \leq ... \leq p_n$ if and only if $s_1 \leq s_2 \leq ... \leq s_n$ and $\frac{p_1}{s_1} \leq \frac{p_2}{s_2} \leq \ldots \leq \frac{p_n}{s_n}$ hold. The elements of the sequence $s_1, s_2, ..., s_n$ in this case are integers that denote the numbers of allocated mandates, whereas the terms of the sequence $p_1, p_2, ..., p_n$ denote the numbers of populations in respective member countries of the European Union.

An essential part of the quoted article 9A are so-called boundary conditions that define a minimum and a maximum number of representatives from a given country, and a total size of an assembly. The two former numbers are given by inequalities, however, due to a large number of possible solutions (Łyko, & Rudek, 2013), they are adopted as indicated, especially because the resolution (Lamassoure, & Severin, 2007) explicitly reads that "the minimum and maximum numbers set by the Treaty must be fully utilised to ensure that the allocation of seats in the European Parliament reflects as closely as possible the range of populations of the Member States", thus furthermore confirming such interpretation. As a result, the problem of allocating seats in the European Parliament can be considered as a degressively proportional distribution problem subject to boundary conditions: $m = 6$, $M = 96$ and $H = 751$, where $m$ denotes the number of representatives from the least populous state of the European Union, $M$ – the number of representatives from the most populous country, and $H$ – the number of all members of the European Parliament (Dniestrzański, 2014; Dniestrzański, & Łyko 2014; Serafini, 2012; Felgado-Marquez, Kaeding, & Palomares, 2013; Grimmett et al., 2011).

## 2. Boundary conditions of a degressively proportional distribution

Interestingly enough, the boundary conditions significantly influence both the likelihood of finding a distribution as well as the number of feasible solutions, when the problem is not inconsistent (Łyko, 2013; Dniestrzański, & Łyko, 2014). Therefore, a feasibility analysis of specific boundary conditions should precede any political discussions, and consequently, legal regulations. For that reason, an answer to a question which triples of natural numbers ($m$, $M$, $H$) can produce a system of boundary conditions for a degressively proportional distribution becomes important. In such a case we often say that a triple ($m$, $M$, $H$) is not an inconsistent system of boundary conditions. Unfortunately, such reasoning must not ignore the elements of the sequence $p_1, p_2, ..., p_n$. One cannot expect just one universal answer. Feasibility or inconsistency of a given system of boundary conditions ($m$, $M$, $H$) depends on the sequence $p_1, p_2, ..., p_n$ that determines the allocation.

It is easy to find such numbers $p_1, p_2, ..., p_n$ whose sole degressively proportional distribution is the one with $m = M$, so the elements of the sequence $p_1, p_2, ..., p_n$ are constant. An obvious trivial example is the sequence $p_1 = p_2 = ... = p_n$. However, this is not the only case. More such sequences can be obtained analyzing the sequence of quotients $\frac{p_{i+1}}{p_i}$ of the sequence $p_1, p_2, ..., p_n$, for $i = 1, 2, \dots, n - 1$. This is because the inequalities significantly reduce the possibilities of larger values of elements of the sequence $s_1, s_2, ..., s_n$.

Indeed, the inequality $\frac{p_i}{s_i} \le \frac{p_{i+1}}{s_{i+1}}$ results in $s_{i+1} \le s_i \frac{p_{i+1}}{p_i}$ and $\frac{s_{i+1}}{s_i} \le \frac{p_{i+1}}{p_i}$. The sequence $s_i$ is a nondecreasing sequence of natural numbers, whose minimum element equals $s_1 = m$. Accordingly, if $\frac{p_{i+1}}{p_i} < \frac{m+1}{m}$ holds for every $i$, then $s_1 = s_2 = \dots = s_n = m$.

As a result, the only degressively proportional distributions possible in this case are those with $m = M$ and $H = knm$, where $k$ is any natural number. In other words, the only system of boundary conditions generating a degressively proportional distribution for the above given sequences $p_1, p_2, ..., p_n$ is $(m, m, knm)$. It is also worth noting that the difference between $p_1$ and $p_n$ can be quite large. If the elements of the sequence $p_i = p_1 q^{i-1}$ make a geometric sequence with $q < \frac{m+1}{m}$, then $p_n$ is approximately equal $p_1 q^{n-1}$. In case of constraints adopted for the distribution of mandates to the European Parliament, i.e. when $m = 6$, $n = 28$ and and $q = 1{,}165 < \frac{7}{6}$, the value of $p_{28}$ will be more than sixty-one times greater than the value of $p_1$. Demographic data about populations of member states in the European Union in 2011 show that the ratio between the size of Germany, the most populous member state, and the size of Malta, the least populous country, is approximately 197. For Romania, ranked seventh by population, this ratio is smaller than 61. On the other hand, it is obvious that there exist nontrivial, degressively proportional allocations with $m < M$, so we see that there are no universal boundary conditions and our analysis should always be carried out for a given specific sequence $p_1, p_2, ..., p_n$.

From a practical point of view, the most important part of determining the boundary conditions is the choice of $m$. When deciding about the allocation of seats in collegial bodies, this task is equivalent to the establishing of a minimal and maximal representation. Hence we shall first consider the cases when a value $s_1 = m$ is arbitrarily chosen for a given sequence $p_1, p_2, ..., p_n$. In this situation it is easy to specify the minimum and maximum values of the remaining elements in a triple of boundary values, i.e. $M_{\min}$ and $M_{\max}$, along with $H_{\min}$ and $H_{\max}$. A constant sequence $s_1 = s_2 = \dots = s_n = m$ of course is degressively proportional with respect to any sequence $p_1, p_2, ..., p_n$, therefore $M_{\min} = m$ holds, and $H_{\min} = nm$. The values $M_{\max}$ and $H_{\max}$ for a given sequence $p_1, p_2, ..., p_n$ and parameter $m$, are established by a degressively proportional sequence with maximum values defined recursively: $s_1 = m, s_{i+1} = \left\lfloor s_i \frac{p_{i+1}}{p_i} \right\rfloor$, where $i = 1, 2, \dots, n - 1$. Then $M_{\max} = s_n$, and $H_{\max} = s_1 + s_2 + \dots + s_n$.

It can be easily shown that for a given $m$ there exists such $H$ whose triple $(m, M, H)$ represents boundary conditions of a degressively proportional distribution for all $M \in [M_{\min}, M_{\max}]$. As a result of previous discussion, such $H$ for $M_{\min}$ and $M_{\max}$ are $H_{\min}$ and $H_{\max}$, therefore it suffices to prove that such $H$ can be found for $M \in [M_{\min}, M_{\max}]$. To this end, one has to prove that if $M$ can establish a boundary condition, also $M-1$ is such a number. By assumption, we have $M > M_{\min}$, and as a consequence, for some $H$ there exists at least one degressively proportional distribution $s_1, s_2, ..., s_n$ subject to boundary conditions $(m, M, H)$, that for some $i$, $s_{i+1} = M$ and $s_i < M$ hold. If $i$ is the largest number among $1, 2, \dots, n-1$ with this property, then a sequence $s_1, s_2, \dots, s_i, s_{i+1}-1, s_{i+2}-1, s_n-1$ is degressively proportional with respect to $p_1, p_2, ..., p_n$, satisfying the boundary conditions $(m, M-1, H')$, where $H' = s_1, s_2, \dots, s_{i-1}, s_i - 1, s_{i+1}-1, s_n - 1$.

An analogous situation is when we arbitrarily take the value of $M$, given the sequence $p_1, p_2, ..., p_n$. A constant sequence $M = s_n = s_{n-1} = \dots s_1$ is always degressively proportional, hence $M_{\max} = M$ and $H_{\max} = nM$. A recursively defined sequence $s_n = M$, $s_{i-1} = \left\lceil s_i \frac{p_{i-1}}{p_i} \right\rceil$ for $i = n, n-1, \dots, 2$, satisfies the condition of degressive proportionality for the sequence $p_1, p_2, ..., p_n$, and its elements are the minimum values among all possible degressively proportional sequences with $s_n = M$. If any element would be smaller, then the condition of degressive proportionality would be violated. The values of $m_{\min}$ and $H_{\min}$ are determined, as $m_{\min} = s_1$ and $H_{\min} = s_1 + \dots + s_n$.

Similarly we can also demonstrate that for any $m \in [m_{\min}, m_{\max}]$ there exists such $H$ and a sequence $s_1, s_2, ..., s_n$ degressively proportional with respect to $p_1, p_2, ..., p_n$, whose triple $(m, M, H)$ represents boundary conditions. In this case, it suffices to prove that if $m \in (m_{\min}, m_{\max})$ can define a boundary condition, then also $m + 1$ is such number. Indeed, $m < M$ holds under the given assumptions, therefore there exists at least one sequence $s_1, s_2, ..., s_n$ degressively proportional with respect to $p_1, p_2, ..., p_n$, whose triple $(m, M, H)$ represents boundary conditions for some $H$. Then for any sequence with this property, we can find such $i$ that $s_i = m$ and $s_{i+1} > m$. If $i$ is the smallest number among $1, 2, \dots, n-1$ with this property, then the sequence $s_1 + 1, s_2 + 1, \dots, s_i + 1, s_{i+1}, s_{i+2}, \dots, s_n$ is degressively proportional with respect to $p_1, p_2, ..., p_n$ that satisfies the boundary conditions $(m + 1, M, H')$, where $H'$ is the sum of its elements.

In both cases however, we can find a sequence $p_1, p_2, ..., p_n$ with such $H \in (H_{\min}, H_{\max})$ that a system of boundary conditions is inconsistent, i.e. there does not exist a sequence $s_1, s_2, ..., s_n$ degressively proportional with respect to $p_1, p_2, ..., p_n$, whose triple $(m, M, H)$ represents a system of boundary conditions. Indeed, it suffices to take a sequence $p_1, p_2, ..., p_n$ so that $\frac{p_2}{p_1} > \frac{m+1}{m}$ and $p_2 = p_3 = \dots = p_n$, $n > 2$, Then $s_1 = m$, $s_2 = s_3 = \dots = s_n = m+1 = M$ is a degressively proportional sequence with respect to $p_1, p_2, ..., p_n$ with $H = nm + n - 1$. Yet, there does not exist a sequence $s_1, s_2, ..., s_n$ degressively proportional with respect to $p_1, p_2, ..., p_n$, whose boundary conditions are represented by a triple $(m, M, H-1)$, because the value of $s_1$ cannot be reduced by one, and the decrease of any element among $s_1, s_2, ..., s_n$ requires the decrease of all remaining, thus yielding a sum that is smaller than $H-1$.

Therefore boundary conditions cannot be specified arbitrarily. Firstly, a minimum and a maximum representation, i.e. the values $m$ and $M$ are restricted as above mentioned, and secondly, even if these constraints are fulfilled, the existence of a sequence $s_1, s_2, ..., s_n$ with the sum $H$ satisfying the inequalities $H_{\min} \le H \le H_{\max}$ is not ensured. For $H = H_{\min}$ and $H = H_{\max}$ the sequences $s_1, s_2, ..., s_n$ are of course determined uniquely, yielding either distributions that are closest to proportional allocations or equal distributions. Generally these are the only possible boundary conditions leading to unique solutions. However, they seem unacceptable from a practical point of view. Therefore, we have to seek specific distribution consenting an arbitrary selection or giving additional criteria that allow a unique solution.

### 3. Distribution of mandates in the European Parliament

Table 1 presents populations of the member states of the European Union (as of 1 January 2012, based on Eurostat data, column 2), percentage shares (column 3) and examples of distribution of seats in the European Parliament. Column 4 presents the distribution during the 2014-2019 term, column 7 – a maximum distribution, column 7 – a maximum distribution. Columns 5, 8 and 11 give the numbers of citizens of a given country represented by one member of the EP under a given distribution, and columns 6, 9 and 12 – the percentage shares of mandates allocated to a given country in total number of mandates.

Comparing populations with numbers of mandates allows to examine whether the principle of degressive proportionality is satisfied. The allocation of mandates to the European Parliament among all member countries for the 2014-2019 term was proposed by the Committee on Constitutional Affairs and does not meet the condition of degressive proportionality (see columns 4 and 5 in table 1). This is a consequence of methodology chosen by the Committee. Having in mind previous, historical allocations of seats in the past terms of the European Parliament and the accession of Croatia to the European Union, a distribution of seats was adopted as binding so that no member state loses more than one seat of those allocated in the 2009-2014 term and the distribution is close to a degressively proportional one. However, the adopted report explicitly states that this solution is temporary and that efforts will be made to establish "a durable and transparent system which, in future, before each election to the European Parliament, will allow seats to be apportioned amongst the Member States in an objective manner, based on the principle of degressive proportionality" (Gualtieri, & Trzaskowski 2013).

Under a maximum representation (columns 7-9), it is assumed that the smallest country by population is allocated 6 mandates, then each larger country, in an increasing order, is allocated the largest possible number of mandates, so that the principle of degressive proportionality remains satisfied. The model of maximum representation does not set the limits of mandates allocated to a country or the total number of mandates. Given current populations, the largest country could be allocated almost tenfold the current limit, i.e. $M_{\max} = 902$ mandates. As known, this distribution is close to a proportional allocation that can be seen when we compare the percentage shares of seats allocated to a country in the total number of seats in the EP with the percentage share of its population in the total population of all member states of the EU (columns 3 and 9). This is also confirmed when we compare the extreme values of citizens from a given country represented by one member of the EP (column 8). Under a maximum representation, the difference between these values for the largest and the smallest country by population is smallest among all possible distributions satisfying the principle of degressive proportionality. The total number of seats allocated under this model is $H_{\max} = 5666$ (with $H_{\min} = 6 \cdot 28 = 168$), considerably more than the adopted limit of 751. Modifying this distribution so that the countries which should be allocated more than 96 mandates, get 96 seats, also yields a distribution that satisfies the condition of degressive proportionality, even if most of countries (Austria and states larger by population than Austria) will be allocated the same number of seats. With this modification the size of the European Parliament would be $H_{max} = 1949$.

Under a minimum representation (columns 10–12), the largest country by population is allocated 96 seats, then smaller countries, in a decreasing order, are allocated the smallest possible numbers of mandates, so that the principle of degressive proportionality remains satisfied. Analogously, as before, no limit to the smallest possible number of mandates is introduced. Under this model, the smallest feasible number of mandates is $H_{min} = 666$ (with $H_{max} = 96 \cdot 28 = 2688$).

This distribution clearly reveals that such countries as France, the UK or Spain have less mandates in the 2014-2019 term of the European Parliament than required by the principle of degressive proportionality. It is also helpful in case when additional constraints are introduced, such as that the least populous country has to be allocated 6 mandates, and the total number of seats must not exceed 751. It suffices to modify the distribution in such a way that countries which have less than 6 seats gain additional mandates, i.e. we have to allocate 15 additional seats. Then $H'_{min} = 681$ (see table 2). If the total size is $H = 751$, then it suffices to allocate additional 70 seats, so that the principle of degressive proportionality is satisfied.

**Table 1.** Exemplary allocations of mandates to the European Parliament

| Country | Population | | Number of mandates | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | Current allocation | | | Maximum representation | | | Minimum representation | | |
| | in thousands | % | | in thousands | % | | in thousands | % | | in thousands | % |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| **Germany** | 81 843.7 | 16.1 | **96** | 852.5 | 12.8 | **902** | 90.7 | 15.9 | **96** | 852.5 | 14.4 |
| **France** | 65 397.9 | 12.9 | **74** | 883.8 | 9.9 | **721** | 90.7 | 12.7 | **77** | 849.3 | 11.6 |
| **UK** | 62 989.6 | 12.4 | **73** | 862.9 | 9.7 | **695** | 90.6 | 12.3 | **75** | 839.9 | 11.3 |
| **Italy** | 60 820.8 | 12.0 | **73** | 833.2 | 9.7 | **672** | 90.5 | 11.9 | **73** | 833.2 | 11.0 |
| **Spain** | 46 196.3 | 9.1 | **54** | 855.5 | 7.2 | **511** | 90.4 | 9.0 | **56** | 824.9 | 8.4 |
| **Poland** | 38 538.4 | 7.6 | **51** | 755.7 | 6.8 | **427** | 90.3 | 7.5 | **47** | 820.0 | 7.1 |
| **Romania** | 21 355.8 | 4.2 | **32** | 667.4 | 4.3 | **237** | 90.1 | 4.2 | **27** | 791.0 | 4.1 |
| **Netherlands** | 16 730.3 | 3.3 | **26** | 643.5 | 3.5 | **186** | 90.0 | 3.3 | **22** | 760.5 | 3.3 |
| **Belgium** | 11 290.9 | 2.2 | **21** | 537.7 | 2.8 | **126** | 89.6 | 2.2 | **15** | 752.7 | 2.3 |
| **Greece** | 11 041.3 | 2.2 | **21** | 525.8 | 2.8 | **124** | 89.0 | 2.2 | **15** | 736.1 | 2.3 |
| **Czech Rep.** | 10 541.8 | 2.1 | **21** | 502.0 | 2.8 | **119** | 88.6 | 2.1 | **15** | 702.8 | 2.3 |
| **Portugal** | 10 505.4 | 2.1 | **21** | 500.3 | 2.8 | **119** | 88.3 | 2.1 | **15** | 700.4 | 2.3 |
| **Hungary** | 9 957.7 | 2.0 | **21** | 474.2 | 2.8 | **113** | 88.1 | 2.0 | **15** | 663.9 | 2.3 |
| **Sweden** | 9 482.9 | 1.9 | **19** | 499.1 | 2.5 | **108** | 87.8 | 1.9 | **15** | 632.2 | 2.3 |
| **Austria** | 8 443.0 | 1.7 | **19** | 444.4 | 2.5 | **97** | 87.0 | 1.7 | **14** | 603.1 | 2.1 |
| **Bulgaria** | 7 327.2 | 1.4 | **17** | 431.0 | 2.3 | **85** | 86.2 | 1.5 | **13** | 563.6 | 2.0 |
| **Denmark** | 5 580.5 | 1.1 | **13** | 429.3 | 1.7 | **65** | 85.9 | 1.2 | **10** | 558.1 | 1.5 |
| **Finland** | 5 404.3 | 1.1 | **13** | 415.7 | 1.7 | **63** | 85.8 | 1.1 | **10** | 540.4 | 1.5 |
| **Slovakia** | 5 401.3 | 1.1 | **13** | 415.5 | 1.7 | **63** | 85.7 | 1.1 | **10** | 540.1 | 1.5 |
| **Ireland** | 4 582.8 | 0.9 | **11** | 416.6 | 1.5 | **54** | 84.9 | 1.0 | **9** | 509.2 | 1.4 |

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| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **Croatia** | 4 398.2 | 0.9 | **11** | 399.8 | 1.5 | **52** | 84.6 | 0.9 | **9** | 488.7 | 1.4 |
| **Lithuania** | 3 007.8 | 0.6 | **11** | 273.4 | 1.5 | **36** | 83.6 | 0.6 | **7** | 429.7 | 1.1 |
| **Slovenia** | 2 055.5 | 0.4 | **8** | 256.9 | 1.1 | **25** | 82.2 | 0.4 | **5** | 411.1 | 0.8 |
| **Latvia** | 2 041.8 | 0.4 | **8** | 255.2 | 1.1 | **25** | 81.7 | 0.4 | **5** | 408.4 | 0.8 |
| **Estonia** | 1 339.7 | 0.3 | **6** | 223.3 | 0.8 | **17** | 78.8 | 0.3 | **4** | 334.9 | 0.6 |
| **Cyprus** | 862.0 | 0.2 | **6** | 143.7 | 0.8 | **11** | 78.4 | 0.2 | **3** | 287.3 | 0.5 |
| **Luxembourg** | 524.9 | 0.1 | **6** | 87.5 | 0.8 | **7** | 75.0 | 0.1 | **2** | 262.4 | 0.3 |
| **Malta** | 416.1 | 0.1 | **6** | 69.4 | 0.8 | **6** | 69.4 | 0.1 | **2** | 208.1 | 0.3 |
| **TOTAL** | 508 077.9 | 100 | **751** | | 100 | **5 666** | | 100 | **666** | | 100 |

It is worth mentioning here that, on the one hand, the 'surplus' of 70 mandates results in numerous degressively proportional distributions that satisfy the conditions ($m$, $M$, $H$) = (6,96751), but on the other hand, this exposes the fact that after the accession to the EU of large countries by population, such as Ukraine or Turkey that should be allocated more than 70 mandates, either the size of the European Parliament will exceed 751 or the upper limit will have to be much lower than 96.

In order to allocate the additional mandates, one can employ a sequence that recursively determines the minimum distribution. For example, the second largest country by population (i.e. France) is allocated a greater number of seats, also increasing the numbers of seats for other countries, according to an algorithm based on this sequence $s_{28} = 96$, $s_{27} = s'$, where $s' > 77, s_{i-1} = \left\lceil s_i \frac{p_{i-1}}{p_i} \right\rceil$, for i = 28, 27, …, 2, however, the total size must not exceed 751. If we take $s_{28} = 96$, $s_{27} = 86$, then the total will be 733 (see table 2, column 5). The procedure can go on, trying to increase the number of seats for subsequent, smaller countries (columns 6 and 7).

**Table 2.** Subsequent iterations under minimum representation

| Country | Population | Subsequent iterations under minimum representation | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | | 4 | | 5 | | 6 | | 7 | |
| **Germany** | 81 843.7 | **96** | 852.5 | **96** | 852.5 | **96** | 852.5 | **96** | 852.5 | **96** | 852.5 |
| **France** | 65 397.9 | **77** | 849.3 | **77** | 849.3 | **86** | 760.4 | **86** | 760.4 | **86** | 760.4 |
| **United Kingdom** | 62 989.6 | **75** | 839.9 | **75** | 839.9 | **83** | 758.9 | **85** | 741.1 | **85** | 741.1 |
| **Italy** | 60 820.8 | **73** | 833.2 | **73** | 833.2 | **81** | 750.9 | **83** | 732.8 | **83** | 732.8 |
| **Spain** | 46 196.3 | **56** | 824.9 | **56** | 824.9 | **62** | 745.1 | **64** | 721.8 | **64** | 721.8 |
| **Poland** | 38 538.4 | **47** | 820.0 | **47** | 820.0 | **52** | 741.1 | **54** | 713.7 | **54** | 713.7 |
| **Romania** | 21 355.8 | **27** | 791.0 | **27** | 791.0 | **29** | 736.4 | **30** | 711.9 | **30** | 711.9 |
| **Netherlands** | 16 730.3 | **22** | 760.5 | **22** | 760.5 | **23** | 727.4 | **24** | 697.1 | **24** | 697.1 |
| **Belgium** | 11 290.9 | **15** | 752.7 | **15** | 752.7 | **16** | 705.7 | **17** | 664.2 | **17** | 664.2 |
| **Greece** | 11 041.3 | **15** | 736.1 | **15** | 736.1 | **16** | 690.1 | **17** | 649.5 | **17** | 649.5 |
| **Czech Republic** | 10 541.8 | **15** | 702.8 | **15** | 702.8 | **16** | 658.9 | **17** | 620.1 | **17** | 620.1 |
| **Portugal** | 10 505.4 | **15** | 700.4 | **15** | 700.4 | **16** | 656.6 | **17** | 618.0 | **17** | 618.0 |
| **Hungary** | 9 957.7 | **15** | 663.8 | **15** | 663.8 | **16** | 622.4 | **17** | 585.7 | **17** | 585.7 |
| **Sweden** | 9 482.9 | **15** | 632.2 | **15** | 632.2 | **16** | 592.7 | **17** | 557.8 | **17** | 557.8 |
| **Austria** | 8 443.0 | **14** | 603.1 | **14** | 603.1 | **15** | 562.9 | **16** | 527.7 | **16** | 527.7 |
| **Bulgaria** | 7 327.2 | **13** | 563.6 | **13** | 563.6 | **14** | 523.4 | **14** | 523.4 | **14** | 523.4 |
| **Denmark** | 5 580.5 | **10** | 558.1 | **10** | 558.1 | **11** | 507.3 | **11** | 507.3 | **11** | 507.3 |

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **Finland** | 5 404.3 | **10** | 540.4 | **10** | 540.4 | **11** | 491.3 | **11** | 491.3 | **11** | 491.3 |
| **Slovakia** | 5 401.3 | **10** | 540.1 | **10** | 540.1 | **11** | 491.0 | **11** | 491.0 | **11** | 491.0 |
| **Ireland** | 4 582.8 | **9** | 509.2 | **9** | 509.2 | **10** | 458.3 | **10** | 458.3 | **10** | 458.3 |
| **Croatia** | 4 398.2 | **9** | 488.7 | **9** | 488.7 | **10** | 439.8 | **10** | 439.8 | **10** | 439.8 |
| **Lithuania** | 3 007.8 | **7** | 429.7 | **7** | 429.7 | **7** | 429.7 | **7** | 429.7 | **8** | 376.0 |
| **Slovenia** | 2 055.5 | **5** | 411.1 | **6** | 342.6 | **6** | 342.6 | **6** | 342.6 | **6** | 342.6 |
| **Latvia** | 2 041.8 | **5** | 408.4 | **6** | 340.3 | **6** | 340.3 | **6** | 340.3 | **6** | 340.3 |
| **Estonia** | 1 339.7 | **4** | 334.9 | **6** | 223.3 | **6** | 223.3 | **6** | 223.3 | **6** | 223.3 |
| **Cyprus** | 862.0 | **3** | 287.3 | **6** | 143.7 | **6** | 143.7 | **6** | 143.7 | **6** | 143.7 |
| **Luxembourg** | 524.9 | **2** | 262.4 | **6** | 87.5 | **6** | 87.5 | **6** | 87.5 | **6** | 87.5 |
| **Malta** | 416.1 | **2** | 208.1 | **6** | 69.4 | **6** | 69.4 | **6** | 69.4 | **6** | 69.4 |
| **TOTAL** | 508 077.9 | **666** | | **681** | | **733** | | **750** | | **751** | |

This approach is not able however to resolve the known problem when the selection of a distribution is not unique (Cegiełka at al. 2010; Dniestrzański 2013; Słomczyński, & Życzkowski). For instance, instead of allocating 86 seats to France, as in our example, France might be allocated 80 seats, or any number between 77 and 86, and smaller countries might be allocated more. In any case, using this algorithm guarantees that the principle of degressive proportionality is satisfied.

## 4. Summary

Due to the lack of unambiguous indications regarding the methods of degressively proportional distribution of goods, there emerge various interpretations of provisions of the Lisbon Treaty. As a result, it becomes significant to find the acceptable solutions. This leads to an analysis of boundary conditions of a degressively proportional distribution. It turns out that defining a minimum and a maximum representation is subject to the smallest constraints. For every sequence, and associated degressively proportional allocation, one can determine a minimum or maximum allocation, assuming either of them. In addition, any number from resulting intervals can be considered a boundary condition. However, this is not valid as regards the size of the assembly. One can define the minimum and maximum values subject to a minimum or maximum representation, but some numbers from the respective intervals cannot be elements of boundary conditions. What is more, determining such numbers seems a computationally complex problem.

There are no indications concerning the number of feasible solutions. Apart from trivial cases when allocation is unique, it is difficult to find this number. It is known however, that for large $n$, under a certain system of boundary conditions, the set of feasible solutions cannot be searched in a manageable time. Yet, some simulations are possible if restrictions on boundary conditions are known. Such simulations can lead to establishing an additional rule that points towards a unique solution.

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