European Proceedings of
Computers and Technology
EpCT
www.europeanproceedings.com
e-ISSN: 2672-8834

DOI: 10.15405/epct.23021.43

HMMOCS 2022
International Workshop "Hybrid methods of modeling and optimization in complex systems"

SLACK BASED MODEL FOR ENTERPRISES' EFFICIENCY IMPROVEMENT

M. Pokushko (a)*, I. Medina-Bulo (b), R. Kuzmich (c), R. Pokushko (d), M. Karaseva (e), N. Dubovik (f)
*Corresponding author

(a) Siberian Federal University, Krasnoyarsk, Russia, Reshetnev Siberian State University of Science and Technology, Krasnoyarsk, Russia, University of Cadiz, 10, street University of Cadiz, Puerto Real,11519, Spain, mvp1984@mail.ru
(b) University of Cadiz, 10, street University of Cadiz, Puerto Real,11519, Spain, inmaculada.medina@uca.es
(c) Siberian Federal University, Krasnoyarsk, Russia, romazmich@gmail.com
(d) Siberian Federal University, Krasnoyarsk, Russia, pokushko@mail.ru
(e) Reshetnev Siberian State University of Science and Technology, Krasnoyarsk, Russia, Siberian Federal University, Krasnoyarsk, Russia, karaseva-margarita@rambler.ru
(f) Siberian Federal University, Krasnoyarsk, Russia, inbox@ndubovik.ru

Abstract

The paper proposes to improve the enterprises' efficiency of the heat supply system based on Data Envelopment Analysis (DEA). This is a non-parametric method of mathematical programming. The DEA increases the efficiency of the enterprises under study by optimizing the use of resources, as well as minimizing environmental damage from production. The study considers the Slack Based Model (SBM) of the DEA mathematical programming method. The peculiarities of its application are described. It covers main differences in other DEA models. The proposed model is applied to enterprises of the heat supply system. Combined heat and power plants (CHP) are enterprises of the studied sample. In the course of the study, inputs and outputs indicators of the enterprises of the studied sample were optimized in accordance with the main trends in the environmental safety of such enterprise's operation. The CHPP efficiency is calculated in accordance with the minimization of harmful substances emissions into the environment. Experiments were carried out for calculating efficiency indicators applying the SBM model of the DEA method. Calculations for the Charnes, Cooper and Rhodes model (CCR) and Banker, Charnes and Cooper (BCC) models are also presented for comparison. It also presents calculations of inputs and outputs indicators to achieve maximum enterprises' efficiency of the studied sample.

Keywords: Data Envelopment Analysis, Slack Based Model (SBM), heat supply system, minimization of harmful substances emissions, combined heat and power plant

https://doi.org/10.15405/epct.23021.43
Corresponding Author: M. Pokushko
Selection and peer-review under responsibility of the Organizing Committee of the conference
eISSN: 2672-8834

1. Introduction

Recently, it has become quite important to increase the enterprises' efficiency in view of the global economic and resource crisis. The application of methods for improving efficiency in the fuel and energy complex is of particular relevance in the context of the energy resources shortage. Organizations and firms of this field are becoming particularly significant and require close attention to improve the efficiency of such enterprises. The study proposes to increase efficiency of heat supply enterprises based on Data Envelopment Analysis (DEA). This is a non-parametric method of mathematical programming.

2. Problem Statement

Mathematical programming techniques are gaining popularity during the global crisis due to the possibility of optimizing processes at the enterprises under study. The world's limited resources, high competition, rising production costs are forcing managers to introduce new or improved methods to improve efficiency. The application of information technology in production processes is gaining momentum in the modern world of digital achievements. This solves the problems of reducing the production value and saving production costs. Moreover, global problems of environmental degradation are forcing large enterprises to apply methods for improving efficiency. It also minimizes harm to the environment. We propose to apply the DEA method to solve these problems. The DEA increases the efficiency of the enterprises under study by optimizing the use of resources, as well as minimizing environmental damage from production.

3. Research Questions

The paper considers the Slack Based Model (SBM) of the DEA mathematical programming method. The peculiarities of its application are described. It covers main differences in other DEA models. The proposed model will be applied to enterprises of the heat supply system. Combined heat and power plants (CHP) are taken as enterprises of the studied sample. The considered model will be applied to estimate efficiency of these enterprises. The inputs and outputs indicators for the enterprises of the studied sample will be investigated and optimized in accordance with the main trends in the environmental safety of their operation. The CHPP efficiency will be calculated in accordance with the minimization of harmful substances emissions into the environment. Experiments were carried out for calculating efficiency indicators applying the SBM model of the DEA.

4. Purpose of the Study

The objective of the study is to improve efficiency indicators of the heat supply system.

The purpose will be achieved applying a SBM method of mathematical programming DEA to improve the efficiency of CHPPs while optimizing inputs and outputs of enterprises in the sample under study in accordance with the main trends in the environmental safety of their operation. The CHPP efficiency will be calculated in accordance with the minimization of harmful substances emissions into the

https://doi.org/10.15405/epct.23021.43
Corresponding Author: M. Pokushko
Selection and peer-review under responsibility of the Organizing Committee of the conference
eISSN: 2672-8834

environment. Experiments will be carried out for calculating efficiency indicators applying the SBM model. It has been also carried out previously.

5. Research Methods

We described the DEA mathematical programming method in detail in previous studies (Pokushko et al., 2021; Stupina et al., 2021).

The Slack Based Model was introduced by Tone in 2001 (Tone, 2001).

The formula for this model is as follows (Tone, 2001):

$$minimize \quad p = \frac{1-\frac{1}{m}\sum_{i=1}^{m}(s_i^-/s_{iq})}{1+\frac{1}{r}\sum_{i=1}^{r}(s_i^+/y_{iq})} \tag{1}$$

Subject to the following conditions:

$$\sum_{j=1}^{n} x_{ij}\lambda_j + S_i^- = x_{iq}, \text{ i=1, 2, ..., m.}$$

$$\sum_{j=1}^{n} y_{ij}\lambda_j - S_i^+ = y_{iq}, \text{ i=1, 2, ..., r.}$$

$$\lambda_m, S_j^+, S_i^- \geq 0.$$

It is required to supplement the following condition $e^T\lambda = 1$ to the formula to return the variable to the scale. Thus, the efficiency ratio of the SBM is always less than or equal to the efficiency index of the input-oriented Charnes, Cooper and Rhodes model (CCR) (Charnes et al., 1978). This means that Decision Making Unit (DMU) rated efficient by the SBM model is also efficient according to the CCR model.

Superefficiency models are applied for additional analysis of sufficient DMUs. this model can estimate the efficiency of effective units unlike the CCR and Banker, Charnes and Cooper (BCC) models (Banker et al., 1984). It can be obtained as follows (Tone & Tsutsui, 2009). After removing the population from DMU_q, the object under study seeks to find a virtual DMU^* with inputs X* and outputs Y*. It will be the SBM efficiency after such removal. The overefficiency indicator is defined as a difference between the inputs and outputs of both objects under study: DMU* and DMU_q. This difference is given as a variable δ (Charnes et al., 1992). Thus, the super SBM model can be presented as follows (Zhu, 2001):

$$minimize \quad \delta = \frac{1-\frac{1}{m}\sum_{i=1}^{m}(x_i^*/x_{iq})}{1+\frac{1}{r}\sum_{i=1}^{r}(y_i^*/y_{iq})} \tag{2}$$

Subject to the following conditions:

$$\sum_{j=1,\neq q}^{n} x_{ij}\lambda_j + S_i^- = x_{iq}, \text{ i=1, 2, ..., m.}$$

$$\sum_{j=1,\neq q}^{n} y_{ij}\lambda_j - S_i^+ = y_{iq}, \text{ i=1, 2, ..., r.}$$

$$x_i^* \geq x_{iq}, \text{ i=1, 2, ..., m.}$$

$$y_i^* \geq y_{iq}, \text{ i=1, 2, ..., r.}$$

$$\lambda_j, S_i^+, S_i^-, y_i^* \geq 0.$$

https://doi.org/10.15405/epct.23021.43
Corresponding Author: M. Pokushko
Selection and peer-review under responsibility of the Organizing Committee of the conference
eISSN: 2672-8834

An input-oriented super SBM can be obtained from formula (2) with a denominator equal to 1. The higher an efficiency score, the more efficient the DMU under study.

Now present its peculiarities in comparison to other models. the SBM model simultaneously takes into account both input surpluses and output deficiencies and measures efficiency in the range from 0 to 1 unlike the basic CCR and BCC models. Also, a SBM model, in practice, identifies more inefficient objects than, for example, the basic CCR and BCC models for solving certain problems. The main difference between the ADD model and SBM is that the Additive DEA model (ADD) is an absolute measure (a summation of slacks), while SBM calculates an efficiency measure based on average (Cooper, 2014). The measure is invariant to the units of measurement for the analyzed inputs and outputs of sample objects (Cooper et al., 2007).

Having presented basic mathematical descriptions of the SBM model, consider move an experimental part.

The authors define input and output parameters to estimate efficiency of the SBM model. The study conducts efficiencies for the DEA CHP method. This increases the efficiency of the enterprises under study by optimizing the use of resources, as well as minimizing environmental damage caused by such production. We make calculations, as mentioned above, applying a SBM model. Available thermal capacity of the equipment (Gcal/h) - X1, and consumption of standard fuel for the supplied heat energy (thousand tce/year) - X2 were selected as input indicators. Supply of heat energy to the network (thousand Gcal) - Y1, and emission mass (thousand tons per year) - Y2 were selected as output indicators. A study sample includes 16 DMUs.

6. Findings

The SBM model will help to determine efficiency indicators for the CHP sample under study (Cheng, 2014).

Table 1 presents CHP efficiency indicators calculated by a SBM model. It also presents CCR and BCC for comparison.

Table 1. CHP efficiency indicators calculated by the SBM, CCR and BCC model

№ CHP	Ef. SBM	Ef. BCC	Ef. CCR
1	0.996	1.000	0.996
2	1.000	1.000	1.000
3	0.958	0.977	0.977
4	1.000	1.000	1.000
5	1.000	1.000	1.000
6	0.884	0.975	0.975
7	0.987	0.993	0.993
8	0.998	1.000	0.998
9	0.899	0.988	0.988
10	0.983	1.000	0.983
11	0.981	1.000	0.981
12	0.967	1.000	0.967
13	0.920	0.999	0.999

https://doi.org/10.15405/epct.23021.43
Corresponding Author: M. Pokushko
Selection and peer-review under responsibility of the Organizing Committee of the conference
eISSN: 2672-8834

14	0.799	0.945	0.945
15	0.988	1.000	0.988
16	1.000	1.000	1.000

Here:

Ef. is an efficiency indicator.

Table 2 proves that CHP No.2, CHP No.4, CHP No.5, CHP No.16 operate with maximum efficiency. Their efficiency scores are the highest at 1. CHP No.14 is of the least efficiency. Its performance score is 0.799. For example, Table 1 also presents efficiency indicators calculated by the CCR and BCC models. It is clear that some enterprises are less efficient when calculating the SBM model. Therefore, the proposed model will make it possible to give more efficient settings in terms of inputs and outputs to achieve maximum efficiency.

Now, calculate inputs and outputs of each enterprise in order to achieve maximum efficiency applying the SBM model.

Table 2 reflects inputs and outputs indicators calculated by the SBM model to achieve a maximum efficiency indicator of 1.

Table 2. SBM calculated inputs and outputs for achieving a maximum efficiency score of 1.

№ CHP	Y1	Y2	RM, SM (Y1)	RM, SM (Y2)	X1	X2
1	3875.429	13.840	15.429	-0.016	1420	673
2	1378.000	10.000	878.574	-	759	392
3	4198.836	13.196	275.862	-2.153	1234	721
4	4194.000	13.000	-	-	1423	751
5	1180.000	11.000	-	-	725	368
6	3740.426	13.196	435.426	-2.065	1345	567
7	3875.535	12.987	50.535	0.053	1345	681
8	1246.605	8.017	2.605	-0.017	712	308
9	3730.119	12.090	511.085	-1.978	1468	525
10	4053.685	13.051	67.685	-0.051	1345	621
11	1752.416	11.097	33.416	-0.097	697	343
12	2993.848	12.818	99.848	-1.784	1267	423
13	3672.227	12.260	293.227	-0.260	1489	565
14	1506.276	10.255	302.276	-1.255	712	330
15	2955.901	12.678	35.901	-1.665	1258	456
16	3187.000	13.000	-	-	1408	435

Here:

RM is Radial movement,

SM is Slack movement.

Thus, according to Table 2 we can see that for efficient CHPs such as CHP No. 2, CHP No. 4, CHP No. 5, CHP No. 16 inputs and outputs are the same. Their efficiency rate was the highest possible as they operate at maximum efficiency. Improvements for such CHP plants are not required. Table 2 presents input and output indicators to achieve maximum efficiency of 1 for all other CHPPs of the study sample.

Therefore, for example, consider CHP No. 1. It is required to increase the supply of thermal energy to the network by 15.429 thousand Gcal and reduce the emission mass by 0.016 thousand tons per year in order to achieve maximum efficiency of 1. The indicators of the available thermal capacity of the equipment and the equivalent fuel consumption for the supplied heat energy remain unchanged.

7. Conclusion

The purpose has been achieved.

The paper presented the application of the SBM method of mathematical programming DEA to improve the efficiency of CHPPs while optimizing inputs and outputs of enterprises in the sample under study in accordance with the main trends in the environmental safety of their operation. CHP efficiency indicators were calculated in accordance with the minimization of harmful substances emissions into the environment. Moreover, experiments were carried out for calculating efficiency indicators applying the SBM model that was carried out previously. Calculations of inputs and outputs indicators are presented to achieve the maximum enterprises efficiency in the studied sample.

Acknowledgments

This work was supported by the Ministry of Science and Higher Education of the Russian Federation (Grant № 075-15-2022-1121).

References

Banker, R., Charnes, A., & Cooper, W. (1984). Some models for estimating technical and scale inefficiencies in Data Envelopment Analysis. *Management Science, 30*(9), 1078-1092. https://doi.org/10.1287/mnsc.30.9.1078

Charnes, A., Cooper, W. W., & Rhodes, E. (1978). Measuring the efficiency of decision-making units. *European Journal of Operational Research, 2*(6), 429-444. https://doi.org/10.1016/0377-2217(78)90138-8

Charnes, A., Haag, S., Jaska, P., & Semple, J. (1992). Sensitivity of efficiency classifications in the additive model of data envelopment analysis. *International Journal of Systems Science, 23*(5), 789-798. https://doi.org/10.1080/00207729208949248

Cheng, G. (2014). *Data Envelopment Analysis: Methods and MaxDEA Software.* Intellectual Property Publishing House Co. Ltd.

Cooper, W. W. (2014). Origin and Development of Data Envelopment Analysis: Challenges and Opportunities. *Data Envelopment Analysis Journal, 1*(1), 3-10. https://doi.org/10.1561/103.00000002

Cooper, W. W., Seiford, L. M., & Tone, K. (2007). *Data envelopment analysis: A comprehensive text with models, applications, references and DEA-solver software*. Springer. https://doi.org/10.1007/978-0-387-45283-8

Pokushko, M., Stupina, A., Medina-Bulo, I., Dresvianskii, E., Stupin, A., Kuzmich, R., Ruiga, I., & Korpacheva, L. (2021). Evaluating the efficiency of heat and power systems by the data envelopment analysis method. *WSEAS Transactions on Power Systems, 16,* 185-194. https://doi.org/10.37394/232016.2021.16.19

Stupina, A., Pokushko, M., Shagaeva, O., Medina-Bulo, I., Dresvianskii, E., Kuzmich, R., Antamoshkin, O., & Tsepkova, M. (2021). The influence of the heat and power system on the comfort of the living Environment. *WSEAS Transactions on Power Systems, 16,* 344-353. https://doi.org/10.37394/232016.2021.16.34

https://doi.org/10.15405/epct.23021.43
Corresponding Author: M. Pokushko
Selection and peer-review under responsibility of the Organizing Committee of the conference
eISSN: 2672-8834

Tone, K. (2001). A slacks-based measure of efficiency in data envelopment analysis. *European Journal of Operational Research, 130*(3), 498-509. https://doi.org/10.1016/s0377-2217(99)00407-5

Tone, K., & Tsutsui, M. (2009). Network DEA: A slacks-based measure approach. *European Journal of Operational Research, 197*(1), 243-252. https://doi.org/10.1016/j.ejor.2008.05.027

Zhu, J. (2001). Super-efficiency and DEA sensivity analysis. *European Journal of Operational Research, 129,* 443-455.