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IDENTIFICATION OF DIFFERENTIAL EQUATIONS SYSTEMS WITH VARIOUS INPUT EFFECTS

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Abstract

Nowadays, differential equations and their systems are one of the most preferred ways to represent models of dynamic objects. Objects from different areas are dynamic. Therefore, a great number of methods for dynamical systems identification have been developed. However, as processes become more complex, there exist a need to develop new tools. A change in the input action according to a predetermined law could be one of features for dynamic processes. The paper studies the efficiency of the method based on evolutionary algorithms to identify objects in the form of systems of differential equations with various input effects. Genetic programming and differential evolution are the algorithmic basis of the method. The method performs self-configuring of parameters for evolutionary algorithms. The presented paper studies efficiency of the proposed method on five problems where the initial objects are represented by systems of differential equations of various orders. The conducted study takes into account the presence of noise in the initial samples. It demonstrates graphical interpretation of the obtained results. The obtained results prove efficiency of the developed method under various input influences.

Keywords: Differential equation, identification, evolutionary algorithms

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1. Introduction

Mathematical modeling of dynamic systems is an interdisciplinary tool for studying various processes in nature, society and industry. A lot of methods have been developed to simulate dynamic processes. These methods belong to the analytical and numerical, parametric and nonparametric classes (Brester et al., 2020; Ovcharenko, 2020; Roehrl et al., 2020). The study of various systems' behavior often leads to analysis and solution of equations that include characteristics such as a rate of change of system parameters. An analytical representation of such changes are derivatives. They are the basis of differential equations. Differential equations and their systems are the basis for studying in the mathematical modeling field. They are functionally applied in manufacturing. It is extremely important for the development of various branches of science and technology (Brester & Ryzhikov, 2019; Liu et al., 2021).

The theory of differential equations is characterized by its direct application for practical problems (Chu & Marynets, 2021). Differential equations and their systems can be applied as a tool for modeling various phenomena in mechanics, chemical reactions, electrical and magnetic phenomena (Escalante-Martínez et al., 2020). The study of a wide range of problems connected with the strength of materials, biology, economics shows that their solution is reduced to mathematical modeling in the form of a functional dependence described by differential equations and their systems. The representation in the form of differential equations makes it possible to obtain a model that is suitable for further study. In this aspect, it is necessary to take into account the interpretability of the obtained results provided by the symbolic representation of differential equations. It is worth noting that they often contain not one, but several output characteristics when studying manufacturing processes. Systems of differential equations are applied to represent such processes.

The paper analyzes a method based on a self-configuring genetic programming algorithm for identification of dynamic systems with input effects changing according to predetermined laws.

2. Problem Statement

Let it need to be solved the inverse problem of mathematical modelling for a process characterized by an arbitrary number of input and output variables, i.e., build a model in the form of a system of differential equations according to the measurements of inputs and outputs:

$$\begin{cases} y_1^{(k)} = f_1(t, x_1, \dots, x_m, y_1, \dots, y_s) \\ y_2^{(k)} = f_2(t, x_1, \dots, x_m, y_1, \dots, y_s) \\ \dots \\ y_s^{(k)} = f_s(t, x_1, \dots, x_m, y_1, \dots, y_s) \end{cases}$$

where x_{mi} are measurements of input variables, y_{si} are measurements of output variables, m and s are their numbers, respectively, k is an order of the differential equation.

It should be taken into account that input effects can be changed according to the certain laws.

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3. Research Questions

The presented work explores the following research questions:

- What evolutionary methods make it possible to obtain a model of a dynamic object in the form of differential equations and their systems?
- Is it possible to apply the method based on evolutionary algorithms to identify dynamic objects with input effects changed according to certain laws?

4. Purpose of the Study

The purpose of this work is to develop and study a method based on evolutionary algorithms for the dynamic objects identification in the form of differential equations systems. The authors study the efficiency of the approach for various inputs.

5. Research Methods

The paper studies the method, proposed by the authors, for the identification of dynamic objects in the form of differential equations systems. The scheme of the method is presented in Figure 1:

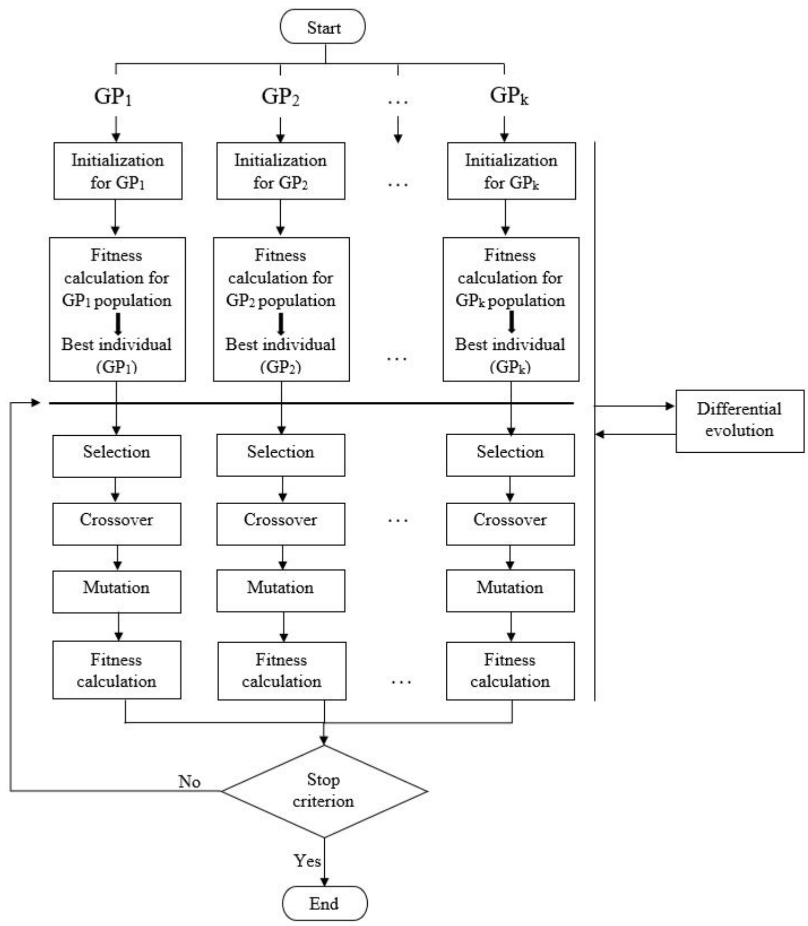

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Figure 1. Algorithm of the evolutionary method for the identification of dynamical systems in the form of the differential equations system

The considered method is characterized by the following features:

1. Differential equations are encoded in the form of trees. The algorithmic basis of the described method is a self-configuring genetic programming algorithm. It is characterized by the representation of the solution in the form of a tree. The authors modified the main evolutionary steps to encode differential equations in the form of a tree. Derivatives were included in the terminal set. The method of differential evolution is applied to optimize numerical constants included in the built differential equation. Thus, a key feature of the method is the automated selection of the structure of the differential equation and numerical coefficients. The method applies algorithms for adaptive configuring of the algorithm's parameters (numerical parameters and parameters with the selection of the type) (Meyer-Nieberg & Beyer, 2007; Storn & Price, 1997). The study of the presented method is given in Karaseva and Semenkin's (2022) study

2. The method was developed to represent the solution in the form of differential equations system. It was based on the parallel run of several genetic programming algorithms. Each algorithm searches for one of the equations of the system (Karaseva & Semenkin, 2022). Fitness is calculated for each individual in each population based on these values in each population; the best individual is determined:

$$fitness = \frac{1}{1 - error},$$

$$error = \frac{\sum_{i=1}^{k}\sum_{j=1}^{n}\left(y_{ij} - \hat{y}_{ij}\right)^2}{sn},$$

where n is sample size, s is number of equations, y_{ij} are values from the original sample, $\hat{y}_{ij}$ is value of the model.

The calculation of fitness (equation) for each individual in each population is carried out by substitution into each of the equations of the system. The best individuals are selected as remaining equations.

Thus, the method helps to encode a system of differential equations of an arbitrary number of equations of an arbitrary order.

6. Findings

Table 1 presents systems of differential equations that were applied to study the considered method.

Table 1. Tasks for testing the method to identify systems of differential equations for various input effects

No	Differential equations systems	Initial sample point
1	$\begin{cases} \frac{dy_1}{dt} = 2y_1 - 5y_2 + 3 \\ \frac{dy_2}{dt} = 5y_1 - 6y_2 - 1 \end{cases}$	$y_1(0)=6$ $y_2(0)=5$
2	$\begin{cases} \frac{dy_1}{dt} = 2y_1 + y_2 \\ \frac{dy_2}{dt} = 3y_1 + te^t \end{cases}$	$y_1(1)=1$ $y_2(1)=2$

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3	$\begin{cases} \frac{dy_1}{dt} = 2y_1 - y_2 - y_3 \\ \frac{dy_2}{dt} = 3y_1 - 2y_2 - 3y_3 + 2t \\ \frac{dy_3}{dt} = 2y_3 - y_1 - y_2 - t^2 \end{cases}$	$y_1(0)=2$ $y_2(0)=3$ $y_3(0)=2$
4	$\begin{cases} \frac{d^2y_1}{dt^2} = -ty_2 \\ \frac{d^2y_2}{dt^2} = y_2 - 2\frac{dy_1}{dt} \end{cases}$	$y_1(1)=4$ $y_2(1)=-4$
5	$\begin{cases} \frac{d^2y_1}{dt^2} = \frac{dy_1}{dt} - \frac{dy_2}{dt} + e^{-t} + \cos t \\ \frac{d^2y_2}{dt^2} = \frac{dy_1}{dt} - \frac{dy_2}{dt} - 2e^t - \sin t \end{cases}$	$y_1(0)=2$ $y_2(0)=0$

The dependences presented in Figure 2 were selected as input effects.

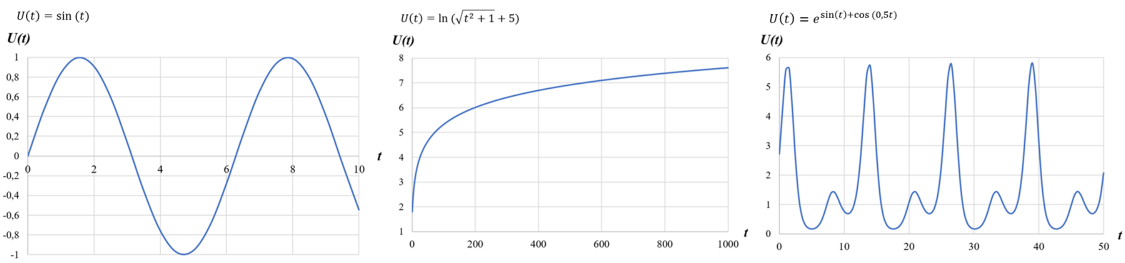

Figure 2. Graph of the input effects

Data with different levels of noise (5%, 10%) were generated based on each system of differential equations and input effects. The sample size for each set was 30 points. Thus, 9 data sets were given for each system. Also, 30 runs were carried out for each set. Table 2 presents error values for the given conditions averaged over 30 runs.

Regression analysis are as follows (Table 2):

Table 2. Testing results of the method with various input effects

Number of the system	Input effects	Error		
		Noise 0%	Noise 5%	Noise 10%
1	$u(t) = sin(t)$	0.053	0.058	0.060
	$u(t) = \ln(\sqrt{t^2+1} + 5)$	0.066	0.072	0.077
	$u(t) = e^{\sin(t)+\cos(0.5t)}$	0.099	0.104	0.107
2	$u(t) = sin(t)$	0.043	0.042	0.044
	$u(t) = \ln(\sqrt{t^2+1} + 5)$	0.001	0.001	0.001
	$u(t) = e^{\sin(t)+\cos(0.5t)}$	0.001	0.001	0.002
3	$u(t) = sin(t)$	0.001	0.001	0.001
	$u(t) = \ln(\sqrt{t^2+1} + 5)$	0.012	0.013	0.012
	$u(t) = e^{\sin(t)+\cos(0.5t)}$	0.001	0.002	0.002
4	$u(t) = sin(t)$	0.026	0.022	0.024
	$u(t) = \ln(\sqrt{t^2+1} + 5)$	0.002	0.002	0.003
	$u(t) = e^{\sin(t)+\cos(0.5t)}$	0.025	0.027	0.031
5	$u(t) = sin(t)$	0.001	0.001	0.001
	$u(t) = \ln(\sqrt{t^2+1} + 5)$	0.001	0.001	0.002

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$u(t) = e^{\sin(t)+\cos(0.5t)}$	0.001	0.001	0.001

Figures 3-7 present the correspondence graphs of the obtained models to the original samples. Green and blue lines indicate outputs of the original objects, red and yellow lines correspond to the outputs of the obtained models (if there are more than two outputs, violet lines are outputs of the original objects and light blue lines are outputs of the obtained models). Complete line overlap with minimal model error is possible.

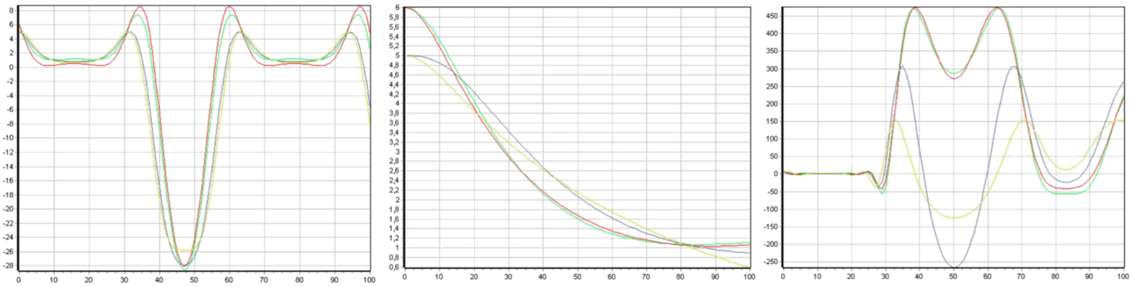

Figure 3. Reaction of object 1 to the described input effects

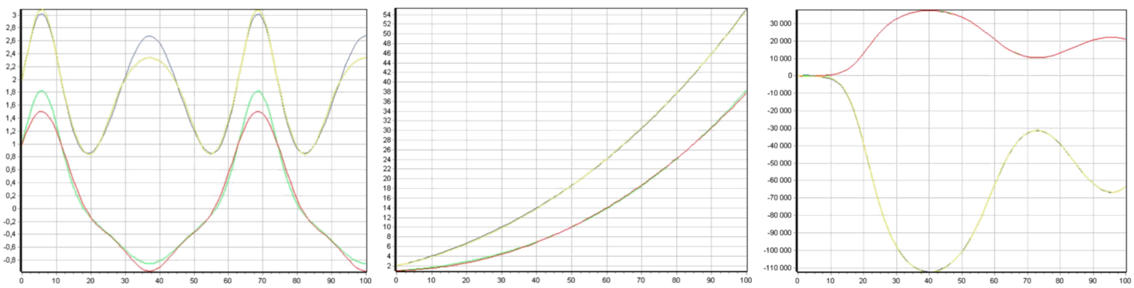

Figure 4. Reaction of object 2 to the described input effects

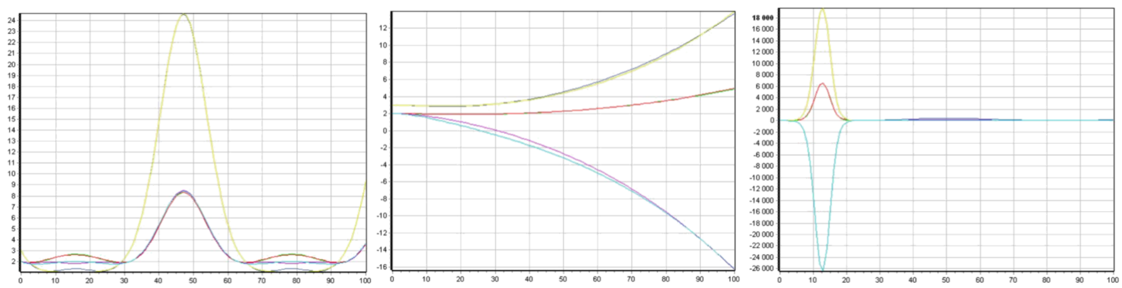

Figure 5. Reaction of object 3 to the described input effects

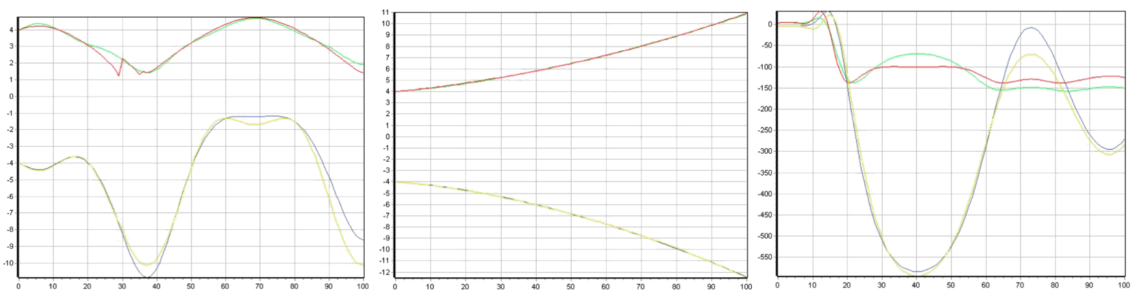

Figure 6. Reaction of object 4 to the described input effects

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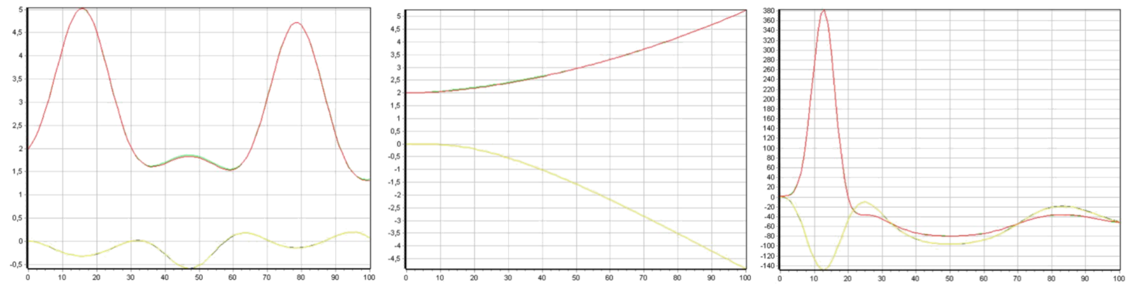

Figure 7. Reaction of object 5 to the described input effects

7. Conclusion

In the course of the work, the study of the quality of identification in the form of differential equations systems for different input influences was conducted. Objects are represented by differential equations systems of various types and orders. The resulting models describe the data well regardless of the type of input effects. It is confirmed graphical interpretation of the obtained values.

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